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## TECHNICAL DOCUMENTATION OF THE LATVIAN LABOUR MARKET MEDIUM AND LONG-TERM FORECASTING AND POLICY ANALYSIS MODEL

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# CONTENT

Introduction	4
1. General overview of the Latvian labour market medium and long-term forecasting and polic	cy
analysis model	/
1.1. Methodological Iranework	/
1.2. Model blocks and logical structure	9 11
2 Mathematical description of the model	13
2.1 Labour demand module	13
2.1.1. Sector labour demand sub-model	13
2.1.2. Occupational labour demand sub-model	19
2.1.3. Modelling labour demand by education	28
2.1.4. Modelling labour demand by gender	36
2.2. Labour supply module	39
2.2.1. Demographic sub-model	39
2.2.2. Education attainment sub-model	51
2.2.3. Working-age population stock-taking sub-model	79
2.2.4. Labour force participation sub-model	92
2.2.5. International migration sub-model	99
2.3. Labour market module: balancing labour demand and supply	108
2.3.1. Labour market sub-model	801 Johon
2.5.1.1. Framework and structure of the labour demand and suppry balancing sub-in	100001
2.3.1.2 Calculation algorithm of vacancies	123
2.3.1.2. Executation algorithm of vacancies	125
2.3.1.4. Declining algorithm of vacancies and occupied working places	129
2.3.1.5. Employment and occupational mobility algorithm	134
2.3.1.6. Synchronization algorithm	164
2.3.2. Wage sub-model	168
3 Model user guide	178
3.1 Model interface	178
3.2. Data input and model undate	181
3.3. Integrated constants in the model	182
3.4. Model output tables	182
4. Description of the results obtained with the model, interpretation and reliability of the result	ts
	186
5. The time frame of the forecasting process	198
61	
References	199
Annex A. The annexes to the body text of the technical documentation	209
Annex A.1. Data specification and sources used in modelling	209
Annex A.2. Period of changes of productivity by sectors	214
Annex A.3. Conformance of the significance of the occupational groups (3-character le	evel)
to criteria by sectors in 2010	214
Annex A.4. Dropout index values	215

Annex A.5. Privileged age structure index values	215
Annex A.6. Constants integrated in the model	216
Annex A.7. Technical description of SDM_IN.XLS fail	225
Annex A.8. Mathematical formula abbreviations	235
Annex A.9. Data recovery time	242
Annex A.10. Data recovery and forecasting time consumption	244

Annex B. Detailed description of the model in the form of the Powersim Studio programming language (separate fail, only in Latvian version of technical documentation)

#### INTRODUCTION

Today, the globalization and information society influences the development of different economic processes. The processes are developing with unprecedented speed and leave an impact both at the regional and global levels. Latvia is the member of the European Union (EU), so it has international, European and local features, including the advanced changes of the situation in economics and labour market.

In the rapidly changing environment the traditional forecasting methods do not allow providing the reasonable forecasts in the average and long-term periods. Time series forecasting methods are intended for short-term forecasting; econometric methods are able to predict the longer term, but only when the degree of impact of influencing factors and form of compliance are not changing in the analysed period. Along with the economic processes the rapid changes and development create a need for economists - practitioners throughout the world to search for new strategic planning tools. In particular, they have been affected by the global economic crisis.

One solution is the broader application of the system dynamics method, including the forecasting of economic processes and labour market processes.

The system dynamics method has been applied in the labour market forecasting in Latvia for a long time. One of the most practicable researches in this field is the study of the University of Latvia (LU) "Research of the long-term forecasting system of the labour market demand and analysis of the improvement opportunities", which has resulted in a dynamic optimization model (DOM). For some time, the Ministry of Economics of the Republic of Latvia (MoE) has used the model results, and then the DOM model has been improved in MoE. With the rapid changes of environment, the previously developed models needs to be supplemented, improved. This document describes the improved model, its principles and operation.

Detailed DOM structure is available in the research published by the University of Latvia and the Ministry of Welfare of the Republic of Latvia.

It is being proposed in the improved model to use optimization theory methods, as well as reduce the application of econometric index in the model, thus transforming the model and, consequently, developing the system dynamics model (SDM) by the principles of system dynamics method.

The first significant improvement of the model is the development of the international labour migration sub-model. Detailed development of this sub-model is based on the passed and ongoing processes in the Latvian labour market, as well as increasing labour migration to more economically developed countries of EU.

In the improved model the decision to abandon environmental and resource module, as well as not to use technology sub-model has been taken. These DOM parts have the low relation to the predictive objects.

The third significant improvement in the model is that the SDM has changed the modelbuilding principles: not only in the demographic sub-model, but in all related labour supply submodels have been set the multi-dimensional system dynamics flows, which provide a more effective reflection of the significant changes and labour amount, forecasting of structure and changes.

The main economic relation changes are related to the use of the production function and changes in productivity. By analysing the new system dynamics researches, it is concluded that the Cobb-Douglas production function almost cannot be applied in the practical researches in macroeconomics. The invested and accumulated capital in the sectors in systems dynamics has been divided into two groups: effective and speculative. Theoretically, system dynamics production function is most often used in effective capital. There is almost no researches, statistics, separating the speculative capital from the effective capital. The effective and speculative capital consolidation and application is causing significant deviations in labour market forecasting, including the short-term and medium-term periods of 3-5 years. Consequently, the SDM base is designed on the basis of Leontief production function.

By improving the model, the number of analysed labour groups has been significantly increased; the forecasting dimensions have been extended. Dimensional comparison of DOM and SDM models are represented in Table 1.

Table 1

Dimension	DOM	SDM	
Sectors	Representation of the economic structure	Statistical classification of economic	
	with the use of the first level of NACE 1.1.	activities of NACE 2 red, the first	
	classification	level of classification	
Occupational groups	Aggregation of 37 occupational groups		
	included in the occupational classification,	Latvian Occupational Classification	
	by combining the similar occupations by	(adapted according to the ISCO-08)	
	experience and skills of the employees,	(3-digit occupation code level) of 127	
	thus forming the labour supply at the	occupational group units.	
	certain education system level and sector		
Education groups	No	Aggregation of levels of education in	
		8 units and 79 fields of education (on	
		the basis of the classification of the	
		Ministry of Education of the Republic	
		of Latvia, third, fourth and fifth groups	
		of the code).	
Gender	No	Men, women	
Age groups	15-19, 20-24, 25-34, 35-44, 45-54, 55-64;	12 ago groups by five years agod from	
	65-74 (the combination of 7 groups by five	15 to 74 years	
	and ten year old age groups)		
Age	No	from 0 to 99 years and one group of	
	NO	population of 100 years old and more	
Economic	No	Economically active population, economically inactive population	
activity	INU		

DOM and SDM dimensions

SDM model provides the forecasts of the following parameters:

- labour demand in terms of economic sectors, skill groups, levels of education;
- population, including the population long-term international migration, in terms of gender, 1-year age groups, skill groups and levels of education;
- economically active population in terms of gender, 5-year age groups, skill groups, levels and fields of education;
- the number of employed population in terms of 5-year age groups, skill groups, levels and fields of education and economic sectors;
- the number of unemployed in 5-year age groups, skills groups, levels of education and sectors of the economy section;
- the number of working places, including free vacancies according to the requirements to gender, occupation, level and field of education determined to employees.

SDM model evaluates the impact of the labour market policy changes on the labour market, including the changes of immigration policy, the number of study places, etc.

In SDM the ergonomics of the model was advanced: one of the model improvement and development principles is the model independence from the operation of the rest of the components - the model in the autonomous mode (or, if needed, with the help of the user/operator) prepare the necessary forecasts from the input data. Calculation of the indexes is not using the sub-model or computer program beyond the system.

In the technical documentation of the Latvian labour market medium and long-term forecasting and policy analysis model the system dynamics Latvian labour market medium and long-term forecasting and policy analysis model is being described.

The first chapter gives a description of the used methodology, the model structure and the logical structure, as well as the data required for modelling.

The second chapter describes the system dynamics model in the mathematical form, indicating and justifying the interaction of the parameters included in the model.

The third chapter represents the detailed model user guide.

The fourth part reflects the description of the results derived from the model.

The fifth part reflects the time curve description of the forecasting process.

SDM model author is Riga Technical University' leading researcher, associate professor Dr.oec.V.Skribans, the project realized by the working group consisting of Dr. oec. V.Skribans (leading expert), Dr. oec. A.Auziņa-Emsiņa, Dr. sc. ing. A.Lektauers.

### 1. GENERAL OVERVIEW OF THE LATVIAN LABOUR MARKET MEDIUM AND LONG-TERM FORECASTING AND POLICY ANALYSIS MODEL

#### **1.1. Methodological framework**

Modelling is one of the ways of problem solution in practice. Modelling is being used when experimenting with the real system or its prototype is too expensive or not possible.

Most common types of analytical modelling are time series, econometric methods etc. These methods provide relatively good forecasting results in the short periods of time when statistically determined connections do not change. Modern economic processes can develop so quickly that these methods could be inapplicable even for 2-3 years of forecasting. On the basis of the research task - to carry out medium and long-term forecasting (up to 30 years) - the provided method groups are considered to be invalid. So long forecasting period is most frequently the system dynamics approach, which determines the choice of the method for model building. The choice of the system dynamics approach is being determined by the compliance of the method with the examined problem, method abstraction and precision level.

By definition, system dynamics analyses the system behaviour in time, depending on the structure of the system elements and their mutual interaction, including cause - effect relationship, feedback links, effects of reaction time etc. Key components of system dynamics are stocks, flows, the system links of the elements that form feedback loops and time delays.

Stocks accumulate materials, non-material objects, represent stocks, stock increasing and reduction potentials (limits). By developing the labour market forecasting model, the task of the stock is to reflect the number of population (including labour, employment, etc.), working positions, study places, etc. in time. The integral calculations are being used for this purpose (see formula 1):

$$LM(t) = LM(t_0) + \int_{t_0}^{T} (TM) dt, \qquad (1)$$

where LM - stock; TM - flow.

As formula 1 shows, the value of the stock size in time t is, primarily, dependent on the stock in the base period (time  $t_0$ ) and, secondly, on the change, determined by the flow in the analysed period. Without the effect (change) of the flow, the stock maintains its initial condition. Here the advantage of this method is evident: if the effect of the influence factors is not known or it cannot be calculated, the stock as the parameter can still be available (it is possible to make calculation on its basis).

By assessing the essence of the method, the main methodological assumption of the labour market forecasting model has been established: the stock (reflecting the number of population (including labour, employed people, etc.), working positions, and study places, etc.) remain unchanged until they are not affected by the influence factors. In practice, it is being expressed as follows: the number of employed (labour volume or the number of working positions, etc.) remains unchanged until it is not being changed by the pre-defined (change) factors. The total influence factors change the stock (repository) through a flow.

Graphically the system dynamic stocks are being marked as rectangle.

The flows reflect the rate of change of the stocks. By developing the labour market forecasting model, the task of the flows is to reflect for how many people per year (or how many

places per year, etc.) the stocks (i.e., the number of population (including labour, employed people, etc.), working positions, study places, etc.) are changing (increasing or decreasing). For calculation of the flows the algebraic calculations are being used.

Graphically system dynamic flows are being marked with double lines or double arrows. The graphic example of stocks and flows is shown in Figure 1.1.



Figure 1.1. Example of stock and flows

In accordance with the principles of system dynamics, system dynamics (the change over time) is being formed on the basis of the accumulation principle. According to the fact that the system dynamic is being formed when flows are accumulated in the repositories (stocks).

By expressing the market forecasting model in practice, the system has been developed, in which the population (labour, employed people, etc.) are being analysed in different stocks (logically related to the status of the population, such as the stocks of "job seekers" reflects the job-seekers, etc.) And in this system the population leave and / or enter, change the stock, using the predetermined flows (e.g., job-seekers from the stock "job seekers" can move to stock "employed" etc.). By comparing to traditional forecasting methods, the model parameters are not calculated as a function of the influencing parameters, but are defined as flow accumulative result.

The system dynamics uses the auxiliaries to ensure the stock - flow system. They are transmitting and processing information and activities from the elements of one system to other (for example, the auxiliary on the basis of stock can select an individual stock element in order to transfer them to other auxiliary or flows, etc.). Auxiliaries integrate stocks (with the associated flows) into a single system; create links between the system elements. The complex of connections of system elements may form the feedback loops, which is also one of the most important system elements.

The connections created by auxiliaries in the system dynamics diagrams are represented by simple arrows (Figure 1.2).

Feed-back loops are being divided into positive and negative loops. The positive feedback loop is defined as a chain, which elements changes and contributes to the further changes of the element itself in the initial direction. If the system has only positive feedback loops, it creates an exponential increase of system elements. None process in the real world by the exponential law can grow indefinitely, it can happen only in a limited time. Exponential increase of system dynamics in the system is being limited by the negative feedback loops. Negative feedback loops are defined as chains which have the element reactions contrary to the effect of the initial element. This definition of negative loops is usually interpreted as follows: changes in one element are transferred around the circle and return to the same element in order to change the element in the direction contrary to the initial changes. Negative feedback loops are considered to be beneficial as they do not allow the systems to collapse in the result of external factors.

Most often the inflation spiral is being mentioned as an example of the positive feedback loops, which are presented in the system dynamics explanatory diagram in Figure 1.2.



Figure 1.2. Inflation spiral

As the example of the negative feedback loop the classic "Hare - wolf" system (it is converted to "profit - entrepreneurs" system in entrepreneurship) is being mentioned. With the growing number of hare in "Hare - wolf" system, the number of wolfs is also growing, which is indicated by the positive loop. But the growth of the number of hares is being reduced by the number of wolfs, as indicated by the negative loop. Explanatory scheme of the "Hare - wolf" system is presented in Figure 1.3.



Figure 1.3. " Hare - wolf" system

As shown in Figure 1.3, the simple model represents a very complex dynamics of the system elements. The number of wolves is being decreased in the model only when the wolves will not have anything to eat; it is being determined by the ratio of hares and wolves. While the number of hares corresponds to the number of wolves, the number of wolves is growing.

Models without feedback loops are not considered as system dynamics models, despite the fact that the method elements can be used here. By forecasting the labour market development, the market feedback loops are taken into account.

Time delays or the rate of exposure response is the next characteristic element of system dynamics. Nowadays, there is an opinion among the system dynamics economists that the delay of economic processes, decisions and response is the main cause of economic instability, including accelerated growth (boom) and recession (crisis).

The real estate market is often mentioned as an example: with the growth in demand, the supply cannot satisfy it momentarily, it takes time to do that, for example, to construct new buildings. System dynamics defines that supply responds to the demand changes with the delay. The consequences of such delays are the following: as long as there are no changes in supply, there is a boom in the real estate market; when there is the change in supply and production surplus - the prolonged crisis in the sector occurs. The delay is observed in various areas and fields: from investment attraction to training of employees in the labour market, etc. Time delay elements are included in the labour market forecasting model.

#### **1.2.** Model blocks and logical structure

The model provides multi-dimensional system dynamics stock and flow system, which reflect the condition and dynamics of workplace, population and other important elements of labour market. Stocks and flows are integrated into algorithms, sub-models and modules. The overall diagram of sub-models and modules is presented in the Figure 1.4.



Figure 1.4. Logic structure of the model

As the Figure 1.4 shows, the model involves three modules - labour supply, demand, and market modules.

**Labour demand module** simulates the labour productivity, labour demand by fields and levels of education, occupations and gender. Labour demand module consists of sector, occupation, education and gender labour demand sub-models.

Sector labour demand sub-model determines the labour productivity and labour demand by sectors on the basis on GDP forecasts.

The labour demand by occupations (in the sub-model of the labour occupational demand) is being calculated on the basis of the sector labour demand.

The labour occupational demand sub-model provides the calculation of the labour demand by levels and fields of education (in the sub-model of the labour education demand), as well as labour demand by gender (in the sub-model of the labour gender demand).

**Labour supply module** simulates the demographic processes, division of the population by age, gender, economic activity, education and occupation. Labour supply module consists of demographic, education attainment, working-age population stock-taking, labour force participation and international migration sub-models.

Demographic sub-model defines population by age groups and genders. This sub-model defines the population fertility, mortality and aging. When the population reaches the 7-year-age, the demographic sub-model defines the number of incoming in the education system population. When the population reaches the working age, demographic sub-model defines the growth of the labour force (in the working-age population stock-taking sub-model), in accordance with previous education (education attainment sub-model) and the estimated economic activity. When the population reaches the retirement age, demographic sub-model defines the decline in labour. The same happens in case of death before the retirement age.

Working-age population stock-taking sub-model represents the labour structure by 5-year age groups, genders, economic activity, education and occupations, that is, reflects the operating results of other sub-models.

Labour force participation sub-model defines the labour structure in the field of economic activity.

Education attainment sub-model defines not only the increase of the primary labour amount (along with demographic sub-model), but also the changes of the labour structure along with education attainment, including lifelong education system.

International migration sub-model defines the change of population and labour along with the international migration processes.

Labour market module simulates employment, unemployment, working positions, vacancies and wages. Labour balancing module consists of two sub-models: labour market and wage sub-models.

Labour market sub-model combines supply and demand, taking into account the working positions, vacancies, the amount of labour and wages.

Wage sub-model analyses the most important processes in the national economy (change of productivity) and labour demand and supply amounts, forming the labour salaries. Labour salaries affect both labour market balance and labour supply formation, that is, affect the choice of education and international migration.

#### **1.3. Data specification and sources**

In model implementation, the main data source is the Central Statistical Bureau (CSB). The data required for the model is presented there, and the sources are indicated (surveys carried out by CSB on the regular basis):

The first source is a *REVIEW OF THE SALARY STRUCTURE* (5-work)<sup>1</sup>.

Review will provide information on employees by gender, age structure, education, occupations, load, type of agreements, number of years worked out in the work position, calculated gross salary (with and without occasional payments), number of paid hours etc.

Next data source is the *REPORT ON WORK PERFORMED*  $(3-work)^2$ .

This will provide statistics on occupied and free working positions by occupational groups and sectors. The report also provides information on the salaries (by occupational groups and sectors).

Similarly, CSB should provide data on the population and its structure, division by gender, oneyear age groups, (*IS06. MEN AND WOMEN AGE STRUCTURE IN THE BEGINNING OF THE YEAR*), data on population mortality, which would be the basis for the calculation of the population mortality indexes (*IM01. AGE AND GENDER STRUCTURE OF DECEASED PEOPLE*), data on the population birth rate, which would be the basis for the calculation of the population birth rate indexes (*ID02. NUMBER OF BORN PEOPLE BY GENDER, ID03. ALIVE BORN PEOPLE BY THE AGE OF MOTHER*), data on long-term migration of the population (*IB04. LONG-TERM AGE AND GENDER STRUCTURE OF MIGRANTS*).

In order to assess the economic development, as well as labour demand changes, data on the sector development and productivity is necessary. It is provided by the gross domestic product by sectors (*IK04. GROSS DOMESTIC PRODUCT BY TYPES OF ACTIVITY*).

Education statistics is being collected by the Ministry of Education and Science. According to the reports, the following information is required: the number of students in general education institutions, by classes and birth years, the graduates of 9 and 12 classes according to *VS-1 GENERAL REPORT OF EDUCATIONAL INSTITUTION (DAY SCHOOL)*<sup>3</sup>, and VV-1 *GENERAL REPORT OF EDUCATIONAL INSTITUTION (EVENING (SHIFT) SCHOOL)*<sup>4</sup>; data on vocational training, number of students, number of dismissed and graduates by source of financing (state-financed, financing from EU structural funds, paid services), by levels of qualification, education programs, gender according to REPORTS OF VOCATIONAL EDUCATIONAL INSTITUTION PROF-2 and PROF-2m<sup>5</sup>.

The data analysis required for the more detailed development of the model is presented in Annex A.1.

<sup>&</sup>lt;sup>1</sup> According to the 06.11.2006 Regulations of the Cabinet of Ministers Nr.922 Annex No.186

<sup>&</sup>lt;sup>2</sup> According to the 06.11.2006 Regulations of the Cabinet of Ministers Nr.922 Annex No.32

<sup>&</sup>lt;sup>3</sup> Draft Regulations of the Cabinet of Ministers "Amendments in 25.08.2008. Regulations of the Cabinet of Ministers No.695 ", Regulations on Education State Statistics Survey Samples" 1.Annex

<sup>&</sup>lt;sup>4</sup> Draft Regulations of the Cabinet of Ministers "Amendments in 25.08.2008. Regulations of the Cabinet of Ministers No.695 ", Regulations on Education State Statistics Survey Samples" 2.Annex

<sup>&</sup>lt;sup>5</sup> Approved by the 20.07.2010 Order No.380 of the Ministry of Education and Science

By observing the requirements of the project while creating the model for average and longterm periods (up to 30 years), the model requires annual data. If it is not possible to apply the period data and MoE labour market forecasts to the certain data, due to non-compliance, in these cases the model should be created on the basis of the previous year or last available year. Data delay will not make problems while defining the basic trends in the average and long-term periods.

In order to make the forecasting scenario analysis, the MoE should prepare forecasts of the long-term development of gross domestic product (added value) by sectors and forecasts of population birth rate and mortality indexes.

#### 2. MATHEMATICAL DESCRIPTION OF THE MODEL

#### 2.1. Labour demand module

Labour demand module is responsible for modelling labour productivity, labour demand by sectors, occupations, education and gender. The module consists of 4 sub-models: sector labour demand sub-model, occupational labour demand sub-model, sub-model of labour demand by education and sub-model of labour demand by gender. Module structure is presented in the figure 2.1.



Figure 2.1. Logical structure of the labour demand module

Figure 2.1 represents that the labour demand by occupations, and further - by education and genders, have been calculated from the sector labour demand in the labour demand module. For the calculation of labour demand in the mentioned dimensions the relevant sub-models have been developed.

Sector labour demand sub-model from the GDP forecasts determines labour productivity and labour demand by sectors.

Calculation of the labour demand by occupations is based on sector labour demand (in the occupational labour demand sub-model).

On the basis of the occupational labour demand sub-model, the labour demand by levels and fields of education has been calculated (in the sub-model of labour demand by education), as well as labour demand by gender (in the sub-model of labour demand by gender).

The further sub-sections describe labour demand module sub-models.

#### 2.1.1. Sector labour demand sub-model

The sub-model is based on Leontief production function, and GDP and employment in the base period, as well as the production function coefficient (further - productivity index) have been used in the calculations. Sub-model logical structure is represented in the Figure 2.2.



Figure 2.2. Logical structure of the labour demand sub-model of national economy sectors

Figure 2.2 represents in a simplified form that the labour demand by sectors is calculated on the basis of GDP forecasts and labour productivity index. Also, the labour productivity index is calculated from GDP forecasting and labour supply by sectors (from the labour supply module), which is necessary in order to ensure the forecasted GDP at the existing labour volume. Labour demand calculation is discussed in detail below, but the calculation logic of the productivity index is represented in the Fig. 2.3.



Figure 2.3. Calculation of productivity index

The necessary labour productivity index is being calculated from GDP forecasts and current employment rate. The index represents the value of the productivity coefficient (Leontief production function) in order to ensure the certain GDP volume at the existing level of employment. Comparing the required productivity index with the actual productivity index, it is possible to determine the changes in capacity, labour productivity (in the model - productivity index) that are necessary to ensure in economics the forecasted GDP, the difference is reflected in the index "necessary changes of productivity index" (the principle that the economy is being developed due to the increasing productivity rather than the employment growth). Productivity index is going slower than the change of required productivity index, as the delay is being affected by the time of productivity changes. Short-term expected productivity index is being used while planning the labour demand for the future period (short-term expected productivity index changes in the current period, while productivity index - only in the future period).

Let us sequentially look at the formation of each sub-model element, starting from the basic principles of calculation of labour productivity and its relation to the Leontief production function.

Leontief production function directly from the volume of output of the sector determines the production factor (labour) demand. Accordingly, along with the changes (with development) in the sector it is possible mathematically to calculate labour changes (development). At the same time, Leontief production function reflects the fact that production factor (labour) demand depends on

both the volume of output and the productivity changes. Unifactor Leontief production function is being determined by the following equation (formula 2):

$$Y_i = A_i \times R_i \,, \tag{2}$$

where

 $Y_t$  - volume of output of the sector;

 $A_i$  - productivity of the production factor i;

 $R_i$  - utilisation of the production factor i.

In order to assess the volume of resource demand, the inverse functions for production functions are being used, i.e., the production resource consumption functions.

In order practically to apply the sub-model, the data of employment and GDP in the base period, as well as the GDP development forecasts (exogenous data of the model) should be available in the forecasting moment. GDP by sectors serves as a measure of sector output volume. Additional exogenous factor is the time of the productivity change (which is constant throughout the forecasting period).

On the ground of the employment in the base period and GDP in the base period, the labour productivity index in the base period is being calculated (formula 3):

$$PK_{N0} = \frac{GDP_{N0}}{N_{N0}},$$
(3)

where

 $PK_{N0}$  - labour productivity index by sectors in the base period; GDP<sub>N0</sub> - GDP by sectors in the base period; N<sub>N0</sub> - employment by sectors in the base period.

After the calculation of the index in the base period the key role belongs to GDP forecasts. On the basis of actual employment and forecasted GDP (exogenous data), the productivity index (formula 4) is being calculated:

$$NPK_{Nt} = \frac{GDP_{Nt}}{N_{Nt}},\tag{4}$$

where  $NPK_N$  - the necessary productivity index by sectors;  $GDP_N$  - GDP forecasts by sectors;  $N_N$  - employment by sectors.

The necessary productivity index determines, which productivity coefficient (Leontief production function) is necessary in order to ensure the certain GDP volume at the current level of employment. Comparing the required productivity index to the actual productivity index, it is possible to determine which changes in labour productivity (in the model - productivity index) are necessary to ensure in economics the forecasted GDP; its difference is presented in the index "necessary changes of productivity index" (in the sub-model the principle that the economy is expanding because of the employment growth, rather than increase in productivity, is observed) (formula 5):

$$NPKI_{Nt} = (NPK_{Nt} - PK_{Nt}) \lor 0, \qquad (5)$$

where

 $NPKI_N$  - the necessary changes of productivity index by sectors;  $NPK_N$  - the necessary productivity index by sectors;  $RK_N$  - productivity index by sectors.

Formula 5 shows that the necessary changes of productivity index are always positive (greater or equal to zero). This indicates that the productivity index (labour productivity) cannot be reduced along with GDP reduction.

Productivity cannot be changed immediately, as soon as the need arises. The actual change of productivity index is slower than the necessary change of productivity index; this delay is affected by the time of productivity change. Calculation of the change of productivity index is presented in formula 6:

$$PKI_{Nt} = \frac{NPKI_{Nt}}{PIL_{N}},\tag{6}$$

where

 $PKI_N$  - changes of productivity index by sectors; NPKI<sub>N</sub> - the necessary changes of productivity index by sectors; PIL<sub>N</sub> - time of the changes of productivity by sectors.

 $1 \text{ Hz}_{\text{N}}$  time of the changes of productivity by sectors.

Time of productivity change is calculated on the basis of the historical data from 1996 to 2010, optimizing the time value so as the historical retrospective would minimize the root-mean-square error of the employment forecast. Equation of root mean square error's calculation is shown in formula 7:

$$S = \sqrt{\sum_{i \in n} \frac{(x_i - \bar{x}_i)^2}{n}} \to \min, \qquad (7)$$

where

S - root mean square error;

x<sub>i</sub> - statistical employment;

 $\bar{x}_i$  - employment calculated in the model;

n - the number of observations.

Time of productivity change is taken as a constant index, which cannot change greatly within 30 years. Justification for this assumption is related to the constancy of the productivity. Productivity in short-term periods is considered to be constant. It is being changed insignificantly in the medium and longer periods. Time of productivity change is the second degree derivative, which changes more slowly, so even during the long-term forecasting it is possible to take it as a constant value. Index values of the time of productivity change are presented in Annex A 2.

The economic meaning of the index "time of productivity change" indicates how quickly the productivity is changing by sectors. If to estimate the national economy as a whole, this index value is 1.205. Higher sector index value than in the overall national economy shows that in the sector the change of the labour productivity is slower than in the national economy. But the smaller sector index value than in the overall national economy shows that the change of the labour productivity is shows that the change of the labour productivity is happening faster than in the national economy.

The index shows the number of years required for full change of productivity in accordance with GDP changes. An index value 1 means that the change of labour productivity is possible within a year. The higher the value of this index, the more important is the role of the sector employment in formation of GDP. For example, in the agriculture, in accordance with EU grants, when the productivity significantly increases, the number of employees decreases - this index is the lowest in the national economy. This indicates that the formation of agricultural GDP has a secondary role for employment, but a key role belongs to productivity growth. However, in the construction sector, where the index is very high, employment has a key role in formation of GDP as the productivity is not changing as fast as it would be necessary. Connection of index with employment is based on a model of logic - if productivity due to the time delay cannot provide the required GDP growth, then this can be done by increasing the number of employees.

Change of productivity index determines the productivity index (formula 8), as well as the expected short-term productivity index (formula 9) (expected productivity index changes in the current period, while the productivity index - only in the future period):

$$PK_{N}(t) = PK_{N}(t_{0}) + \int_{t_{0}}^{T} (PKI_{N}) dt, \qquad (8)$$

where  $PK_N$  - productivity index by sectors;  $RKI_N$  - changes of productivity index by sectors.

Changes of productivity index reflect both the productivity response to changes of GDP and supply effect on productivity:

$$ISPK_{Nt} = PK_{Nt} + PKI_{Nt}, (9)$$

where

 $ISPK_N$  - short-term expected productivity index by sectors;

PK<sub>N</sub> - productivity index by sectors;

 $\ensuremath{\mathsf{PKI}}_N$  - changes of productivity index by sectors.

It is important to emphasize once again that the sub-model involves the limit, after which productivity cannot be reduced simultaneously with the decrease of GDP (i.e., it can only grow at the same time with GDP) (formula 5). If GDP decreases, the productivity index according to this assumption remains constant, but the labour demand will decrease.

Considering that the calculated changes of productivity index do not coincide with the necessary changes of productivity index, changes of GDP cannot fully be ensured by changes of the productivity. Changes of GDP partially make the changes of employment (but the changes that cannot be implemented in the productivity), which make the basis of the changes of labour demand.

In order to model the labour demand, the scheme similar to productivity index is being applied.

Labour demand modelling involves the data on employment in the base period (natively it is being considered that the employment in the base year coincide with the labour demand in the base year), GDP forecasts and short-term expected productivity index (from this sub-model). In addition to an exogenous data - index of the time of employment change has a formal role - in all calculations it is equal to 1, which indicates that during the year it is possible to change employment in accordance with the demand (in this sub-model). Calculation logics of the labour demand forecast is presented in the Figure 2.4.



Figure 2.4. Calculation of labour demand

Figure 2.4 reflects the calculation logics of labour demand. The necessary labour has been determined from the GDP forecasts and short-term expected productivity index values. The index "necessary labour" reflects which should be employment (number of employed persons) in order to ensure the certain GDP volume at the given level of productivity (productivity index in the Leontief production function). The changes of labour are calculated from the required labour - the difference between the required and existing labour. Then the changes of labour are calculated in accordance with the time of employment changes and labour.

Let us investigate the equation of the labour demand calculation algorithm in detail.

In order to determine the necessary labour, GDP forecast and short-term expected productivity index are being used (formula 10):

$$NLS_{Nt} = \frac{GDP_{Nt}}{ISPK_{Nt}},$$
(10)

where

NLS<sub>N</sub> - necessary labour by sectors;

 $GDP_N$  - GDP forecasts by sectors;

ISPK<sub>N</sub> - short-term expected productivity index by sectors.

The index "necessary labour" reflects which should be employment (number of employed persons) in order to ensure the certain GDP volume at the given level of productivity (productivity index in the Leontief production function). The changes of labour are calculated from the required labour - the difference between the required and existing labour (formula 11):

$$NLSI_{Nt} = NLS_{Nt} - LS_{Nt}, \tag{11}$$

where

 $\label{eq:NLSIN} \begin{array}{l} \text{NLSI}_N \text{ - necessary changes of labour by sectors;} \\ \text{NLS}_N \text{ - necessary labour by sectors} \\ \text{LS}_N \text{ - labour demand by sectors.} \end{array}$ 

The changes of the labour demand can be calculated from the necessary changes of labour, formula 12:

$$LSI_{Nt} = \frac{NLSI_{Nt}}{NPIL},$$
(12)

where

 $LSI_N$  - changes of labour demand by sectors; NLSI<sub>N</sub> - necessary changes of labour by sectors; NPIL - time of the change of employment.

As mentioned above, the change of employment time in this sub-model is taken as constant and equals to 1 (one year). This means that within a year it is possible to make the necessary changes of labour in accordance with demand. In this case, the necessary changes of labour coincide with changes of labour demand, but the additional element and additional equation (formula 11) are introduced in order to forecast theoretically the situation when the necessary changes of labour requires a longer period than one year.

Knowing the changes of labour demand, it is possible to calculate the labour demand, formula 13:

$$LS_{N}(t) = LS_{N}(t_{0}) + \int_{t_{0}}^{T} (LSI_{N}) dt, \qquad (13)$$

where

 $LS_N$  - labour demand by sectors.

 $LSI_N$  - changes of labour demand by sectors.

Changes of labour demand reflect both the labour demand response to changes of GDP and supply effect on labour demand (which leads to changes of labour productivity and it, in turn, affect the labour demand).

By combining the productivity index and calculation of labour demand, the sub-model of sectorial labour demand has been developed.

#### 2.1.2. Occupational labour demand sub-model

The sub-model is based on sub-model of labour demand by sectors, as well as on occupations - sectorial statistical data, the ratio of occupations by sectors, and determines the changes of labour demand by occupations - sectors, taking into account the changes of labour demand in the sectors (in relation to changes of GDP in the sector), and structural changes of the occupations on the basis of the target structure.

In order to ensure the operation of sub-model in the forecasting time the functional labour demand sub-model of the sector (with all the incoming indexes and data) should be available, as well as statistical data on occupations - the number of the employed persons in the base period and the target structure of the labour demand by occupations - sectors in the end of the forecasting period.

The sub-model logic of the labour demand by occupations is presented in the Figure 2.5.



#### Figure 2.5. Sub-model logics of labour demand by occupations

As shown in Figure 2.5, the changes of labour demand by occupations and sectors are made of two algorithms: on the basis of the target structure of the changes and on the basis of the demand. The constituent algorithms of the changes are viewed below.

Sub-model has only one stock - "labour demand by occupations and sectors". The calculation equation is presented in formula 14:

$$LS_{PN}(t) = LS_{PN}(t_0) + \int_{t_0}^{T} (LSI_{PN}) dt, \qquad (14)$$

where

LS<sub>PN</sub> - labour demand by occupations and sectors; LSI<sub>PN</sub> - changes of labour demand by occupations and sectors.

Stock initial level is determined from statistical data. Stock changes are determined by the flow of "changes of labour by occupations and sectors" which have been developed by summing the changes associated with structural changes on the basis of the target structure and changes associated with changes in volume of labour demand (formula 15):

$$LSI_{PNt} = LSIMSt_{PNt} + LSINLSI_{PNt},$$
(15)

where

LSI<sub>PN</sub> - changes in labour demand by occupations and sectors;

 $LSIMSt_{PN}$  - changes of labour demand by occupations and sectors in accordance with the structural changes on the basis of the target structure;

 $LSINLSI_{PN}$  - changes of labour demand by occupations and sectors in accordance with the sectorial changes of labour demand.

Calculation scheme of the changes of labour demand by sectors, in accordance with the target structural changes are presented in the Figure 2.6.



Figure 2.6. Calculation of the changes of occupational labour demand by sectors in accordance with the desired structural changes

The first index, affecting the sectorial - occupational separation, is the "changes of labour demand by occupations and sectors in accordance with structural changes on the basis of the target structure". Its calculation algorithm is following: "labour demand by occupations and sectors on the basis of the target structure" is being calculated from the target structure of labour demand by occupations and sectors and labour demand by sectors (from the sub-model of sectorial labour demand) (in formula 16), further, comparing the target and actual labour demand in occupation by sectors, and taking into account the time provided for changes, the changes of labour demand in occupation by sectors are being calculated, taking into account the structural changes on the basis of the target structure (formula 17):

$$LSMSt_{PNt} = LSMSt_{PNt} \times LS_{Nt}, \qquad (16)$$

where

 $LSMSt_{PN}$  - labour demand by occupations and sectors on the basis of the target structure;  $LMSt_{PN}$  - target structure of the labour demand by occupations and sectors;  $LS_N$  - labour demand by sectors.

$$LSIMSt_{PNt} = \frac{LSMSt_{PNt} - LS_{PNt}}{t_b - t},$$
(17)

where

 $LSIMSt_{PN}$  - changes of labour demand by occupations and sectors in accordance with the structural changes on the basis of the target structure;

 $LSMSt_{PN}$  - labour demand by occupations and sectors on the basis of the target structure;

LS<sub>PN</sub> - labour demand by occupations and sectors;

t<sub>b</sub> - forecasting time horizon (forecasting last year);

t - forecasting time (forecasting current year).

It is seen that formula 16 does not require any additional explanations, i.e., a simple multiplication of the target structure by labour demand by sectors. Formula 17 involves the time elements. They help to split the time difference between the actual and target labour demand in the occupation. The target labour demand in the occupation by sectors shows a target level; its difference from the actual labour demand in the occupation, and the sector indicates, which should

be the structure changes throughout the forecasting period. By separating the target changes throughout the whole forecasting period by forecasting period (which is calculated as forecasting last year minus forecasting current year), the required changes by occupations are being calculated within a year. Inclusion of time elements in formula 17 creates the dynamic changes. If there were no other factors affecting the structure, developed algorithm would ensure gradual, smooth transition from actual condition to the target level. If the system involves other factors, the rate of structural changes will be changed.

The other index affecting sectors-occupations distribution is "change of labour demand by occupations and sectors in accordance with sectorial changes of labour demand". This calculation is related to the uneven response to the sectorial changes of labour demand.

In one of the first works on system dynamics "Urban Dynamics", which for the moment has become a classic in this field, Jay Forrester in 1969 developed a separation of labour for workers, specialists and managers. With the development of the economy, the changes of labour demand in each of these groups are different. In the sub-model Forrester principles are taken into account by developing the separation of labour from the three groups to the division of occupations in accordance with Occupational classification of the Republic of Latvia (3-digit level of the occupation code).

One of the modern management concepts considers the organizations as a hierarchical structure, which higher-level elements are based on lower-level elements. This approach may be also transmitted to the labour market, separating by basic occupations (which attract the largest labour amount), specialised occupations (which are ensured by the work performance conditions for workers with basic occupations), management occupations, etc. in the specialty distribution market. The most common hierarchical systems are presented in the form of a pyramid, which is shown in the Figure 2.7.



Figure 2.7. The sample of hierarchical system in the labour market

Figure 2.7 represents the hierarchical sample of the traditional sector, which is based on the elementary occupations, then the specialists, senior specialists and managers are coming. Some sectors do not correspond to this sample, for example, the general staff in education or medicine are specialists or senior specialists, but their work conditions may be ensured by the staff from simple occupational groups. In the improved sub-model on the basis of the statistical data the establishment of the hierarchy of occupations is being proposed, determining the basic occupations that are most commonly found in a particular sector, but the higher level occupations are relatively rare in the analysed sector.

The development of hierarchy of occupations by the staffing is associated with a different response to the growth of the sector and growth of staff demand. The higher the occupation in the hierarchical pyramid is, the smaller are the changes of its representatives (employees), by changing the staff in the sector. And vice versa, the major changes in the sector staff are ensured by occupations, which are most prevailing in the sector and form the basis for a hierarchical pyramid. The increase of the number of employees in the hierarchy is schematically reflected in the Figure

2.8, using one sector as the example - fishing sector (B) in accordance with the NACE classification.



Figure 2.8. Sample of the increase of the number of the sector hierarchy employees for the fishing sector

Figure 2.8 shows by the example of the fishing sector that the growth of the number of the employees is mainly ensured by increase of the lowest level of the hierarchy number of employees.

Management practices shows that not always the growth of the lower level of hierarchy occupations causes an increase in higher groups. Management theory explains that with the means of the administration scale. Every manager has his certain optimal number of subordinate employees and boundaries. With the change in the number of subordinate employees within the optimal boundaries, the number of the managers does not increase. When the number of subordinate employees is beyond the optimal boundaries, the number of managers changes as well. This management concept is also appropriate for labour market in order to forecast occupations. Application of the given concept for forecasting of the occupations in the labour market will ensure dynamism of occupational structure in the sectors.

The development of the changes of occupational labour demand by sectors, taking into account the changes of the sectorial labour demand, is presented in the Figure 2.9.



Figure 2.9. Formation of changes of occupational labour demand by sectors in accordance with the development of the sectors

Figure 2.9 shows that the starting point of the sub-model is the change of labour demand and the structure of occupation in the sector. Changes of labour demand in the sectors are divided

according to the structure of the occupation, by using two algorithms, which are titled as "total changes of labour demand by occupations" and "changes of the major occupations". Changes of occupational labour demand are using one or another algorithm, taking into account the rate of growth of the labour demand.

According to the theoretical assumptions, depending on the sectorial growth of the labour demand growth, the changes of occupational labour demand by sectors, taking into account the changes of the sectorial labour demand, are composed of two affecting indexes: "the total changes of labour demand by occupations" (under conditions of the accelerated growth (boom) of the sectorial labour demand) and "changes of the major occupational labour demand" (under conditions of moderate growth). The equation is presented in formula 18:

$$LSINLSI_{PNt} = \begin{cases} KLSI_{PNt}, \left(\frac{ULSIN_{t}}{2}\right) \in Z\\ SLSI_{t}, \left(\frac{ULSIN_{t}}{2}\right) \notin Z \end{cases},$$
(18)

where

 $LSINLSI_{PN}$  - changes of labour demand by occupations and sectors in accordance with the sectorial changes of labour demand.

ULSIN - estimation of the accumulated changes of labour demand;

KLSI<sub>PN</sub> - total changes of labour demand by occupations and sectors;

SLSI - changes of the major occupations of the labour demand<sup>6</sup>.

The main point demonstrated by formula 18 is how the algorithms of occupational labour demand and structural changes modify due to the growth of labour demand. The index "estimation of the accumulated changes of labour demand" divides the changes of the labour demand in the stages: "paired" and "unpaired". The determination of the stage nature is associated with the belonging of the half of the index "estimation of the accumulated changes of labour demand" to the group of integers. If the index divided by two is an integer, the stage is paired, and vice versa - if the index divided by two is not an integer, the stage is unpaired. The first unpaired stage starts from zero.

The economic and system dynamical essence of this mathematical equation is the following. When the growth level is close to zero the sector extension (growth of labour demand) is associated with the increase in primary occupations: the number of vendors is growing in trade, the number of workers - in the sector, the number of different categories of staff (cleaners, drivers) is not growing, but the existing reserves are being used instead. In the sub-model explanation this stage is denoted as "unpaired". In the further development the reserves disappear. At the stage when the system has no reserves, there is total growth of occupational demand. In the sub-model explanation this stage is denoted as "paired". Then the reserves are created at the paired stage, leading to the end of the paired stage and the start of the unpaired stage. This algorithm can be continued indefinitely, it is possible to be applied both for the labour growth and decreases. Schematically, the development of this process is presented in Figure 2.10.

<sup>&</sup>lt;sup>6</sup> Term is defined in the 23rd formula.



Figure 2.10. Example of gradual development

System dynamic emphasizes the developmental character of the non-linear processes and points out that one of the alternatives and the most probable tendencies is a gradual development.

The index "estimation of the accumulated changes of labour demand" divides the changes of the labour demand (from the index "accumulated changes of labour demand") into stages according to stage length (element "estimation index of the stages of the changes of the labour demand"). The calculation equation is presented in formula 19:

$$ULSIN_{t} = \begin{cases} \left\lfloor \frac{ULSI_{t}}{LSIENK} \right\rfloor, \frac{ULSI_{t}}{LSIENK} > 0\\ \left\lceil \frac{ULSI_{t}}{LSIENK} \right\rceil, \frac{ULSI_{t}}{LSIENK} \le 0 \end{cases}$$
(19)

where

ULSIN - estimation of the accumulated changes of labour demand; ULSI - accumulated changes of labour demand;

LSIENK - estimation index of the stages of changes of labour demand.

The index "estimation of the accumulated changes of labour demand" represents the number of the stage of the changes. The next (current) stage begins when the accumulated increase (change) of labour demand excesses the boundaries of the given stage (boundaries are set by the element "estimation index of the stages of the changes of the labour demand"). The economic and system dynamics essence of this assumption is the following: when the system uses the labour reserve for the certain (basic) occupations, there is only one stage; further, when reserves have been used, and the labour demand is growing in all occupational groups, the next stage begins; formation of reserves causes the beginning of the next stage, etc.

Element "estimation index of the stages of the changes of the labour demand" in the model is used as the constant factor and is equal to 1. This means that the stages of the changes are related to the changes of sectorial labour in the amount of 1%. This means that by changing the sectorial labour demand by 1%, the new stage of the changes begins, where the structure change of occupations is different.

The index "accumulated changes of labour demand" is calculated from the labour demand by sectors and labour demand by sectors in the base period; the calculation is presented in formula 20:

$$ULSI_{t} = \left(\frac{LS_{Nt}}{LS_{N0}} - 1\right) \cdot 100, \qquad (20)$$

where

ULSI - accumulated changes of labour demand;

 $LS_N$  - labour demand by sectors.

 $LS_{N0}$  - labour demand by sectors in the base period.

Accumulated changes of labour demand reflect the changes of labour demand from the beginning of forecasting in the base period. Labour demand and labour demand in the base period are defined in the sub-model of the sectorial labour demand.

The first element affecting the changes of occupational labour demand and occupational structure (from formula 18), i.e., the index "total changes of labour demand by occupations" is calculated in formula 21:

$$KLSI_{PNt} = LSSt_{PNt} \times LSI_{Nt}, \qquad (21)$$

where

KLSI<sub>PN</sub> - total changes of labour demand by occupations and sectors;

 $LSSt_{PN}\xspace$  - structure of labour demand by occupations and sectors;

 $LSI_{\mbox{\scriptsize N}}$  - changes of labour demand by sectors.

The index "total changes of labour demand by occupations" divided changes of labour demand in accordance with the existing occupational structure. This index does not provide structural changes.

Structure of labour demand by occupations in the sector is calculated as the labour demand by occupations and sectors, divided by labour demand in the sector (formula 22):

$$LSSt_{PNt} = \frac{LS_{PNt}}{LS_{Nt}},$$
(22)

where

LSSt<sub>PN</sub> - structure of labour demand by occupations and sectors;

LS<sub>PN</sub> - labour demand by occupations and sectors;

 $LS_N$  - labour demand by sectors.

The second element affecting the changes of occupational labour demand and occupational structure (from formula 18), i.e., the index "changes of the major occupations of the labour demand" is calculated in formula 23:

$$SLSI_{t} = \begin{cases} \frac{KLSI_{PNt}}{SPKI_{Nt}} = \frac{LSSt_{PNt} \times LSI_{Nt}}{SPKI_{Nt}}, LSSt_{Pt} > PSK\\ 0, & LSSt_{Pt} \le PSK \end{cases}$$
(23)

where

SLSI - the changes of the major occupations of the labour demand;

LSSt<sub>PN</sub> - structure of labour demand by occupations and sectors;

PSK - measure of importance of the occupations;

KLSI<sub>PN</sub> - total changes of labour demand by occupations and sectors;

 $SPKI_N$  - the total ratio of the major occupations by sectors;

LSI<sub>N</sub> - changes of labour demand by sectors.

The index "the changes of the major occupations of the labour demand" for the major occupational groups (for those occupational groups, which ratio in the sector is larger than defined in the element "measure of importance of the occupations") determines from the index "total changes of labour demand by occupations" the total changes of the groups of the major occupations by sectors. At the same time, other occupational groups (which do not meet the 'significance" criteria) will not be changed. Instead of the index "total changes of labour demand by occupations" the constituent elements from formula 21 may be used, as presented in formula 23.

The economic essence of the mathematical calculation of the index "the changes of the major occupations of the labour demand" is the following: it determines the changes of the sectorial labour demand only for major occupational groups.

The calculation of the total ratio of the major occupations is presented in formula 24:

$$SPKI_{Nt} = \sum_{i \in P} LSSt_{PNt}^{PSR_i}$$

$$LSSt_{PNt}^{PSR_i} = \begin{cases} LSSt_{Pt}^{i}, LSSt_{PNt}^{i} > PSK^{i} \\ 0, LSSt_{PNt}^{i} \le PSK^{i} \end{cases},$$
(24)

where

 $\begin{array}{l} SPKI_N \mbox{ - the total ratio of the major occupations by sectors;} \\ LSSt_{PN} \mbox{ - structure of labour demand by occupations and sectors;} \\ PSK \mbox{ - measure of importance of the occupations;} \\ P \mbox{ - occupation.} \end{array}$ 

In the index "the total ratio of the major occupations by sectors" are summed up by sector only those occupations whose ratio exceeds the criterion of the significance of the occupational group.

The criterion of the significance of the occupational group is taken as a constant index for all sectors and equals to 3%. This means that the occupations, whose ratio is more than 3%, should be considered as important or primary occupations. The choice of the index is based on Pareto principle and is reasoned by statistical data and analysis (see Annex A3). Despite the low level of the boundary of significance criterion, only a small number of groups of occupations does not confirm to it. But the same occupational groups are employing the highest number of employees in the sectors, which is indicated by the total ratio of the sector (see Annex A3).

In the sub-model the element "labour demand by occupations" has reduced number of dimensions, summing up the labour demand in the occupations by sectors, see formula 25:

$$LS_{Pt} = \sum_{i \in N} LS_{PNt}^{i}, \qquad (25)$$

where

 $LS_{P}$  - labour demand by occupations;  $LS_{PN}$  - labour demand by occupations and sectors; N - sector.

Calculation of the labour demand by occupations in the sub-model has a technical function; this index is not being used in the sub-model.

#### 2.1.3. Modelling labour demand by education

The sub-model of labour demand by education is based on labour demand by occupations, as well as educational - occupational statistical data, educational - occupational compliance matrix, and determines the labour demand by levels and fields of education.

In order to ensure the operation of sub-model in the forecasting time the functional submodel of the sectorial labour demand, sub-model of the occupational labour demand (with all incoming indexes and data) should be available, as well as the following statistical data should be available: 1) data on the number of employees by levels and fields of education of the base year and 2) educational - occupational compliance matrix.

The logic of sub-model is the following: changes of labour demand by occupations are divided into positive (growth) and negative (decrease) changes. The reduction of labour demand by occupation leads to reduction of labour demand in proportion to the existing educational structure (by fields and levels of education). But the increase of the labour demand by occupations leads to the formation of labour demand with an appropriate education. The correspondence of the education to the occupation is being determined by the compliance matrix of the occupation (hereinafter referred to as matrix, educational compliance matrix). Sub-model involves a three-stage algorithm, which, in accordance with the matrix, determines the formation of the labour demand by fields and levels of education. At the first stage the labour demand is being calculated according to the matrix and structure of labour demand by fields and levels of education, at the second stage - in accordance with the matrix and the structure of labour demand by levels of education, and the at the third stage - according to a matrix structure, it is being visually presented in figure 2.11.



Figure 2.11.Logics of the sub-model of labour demand by education

The first mathematical step of the sub-model creating is associated with the division of the changes of labour demand by occupations into positive (greater than zero) and negative (less than zero) changes. The negative changes create the reduction of labour demand by field, levels of education and occupations, but the positive changes create growth. Division of changes into two

parts is associated with an application of different algorithm for every part. Division of changes is presented in figure 2.12.



# Figure 2.12. Transition from the sub-model of occupational labour demand to sub-model of labour demand by education

Figure 2.12 presents that the index "changes of labour demand by occupations and sectors" from the sub-model of occupational labour demand is divided into in the indexes of growth and reduction of labour demand, which will later be used in the sub-model of the educational labour demand.

Equations of division of changes are shown in formulas 26 and 27:

$$LSS_{p_{t}} = \begin{cases} LSI_{p_{t}}, LSI_{p_{t}} < 0\\ 0, LSI_{p_{t}} \ge 0 \end{cases},$$
(26)

$$LSP_{Pt} = \begin{cases} LSI_{Pt}, LSI_{Pt} > 0\\ 0, LSI_{Pt} \le 0 \end{cases},$$
(27)

where

 $LSS_P$  - decrease of labour demand by occupations;  $LSP_P$  - growth of labour demand by occupations;

LSI<sub>P</sub> - changes of labour demand by occupations.

It is important to note that the sub-model of occupational demand does not determine the index "changes of labour demand by occupations". This is a simple summing up of the changes of labour demand by occupations and sectors. The calculation equation is presented in formula 28:

$$LSI_{Pt} = \sum_{i \in N} LSI_{PNt}^{i}, \qquad (28)$$

where

 $\label{eq:LSIP} \begin{array}{l} \text{LSI}_{P} \text{ - changes of labour demand by occupations;} \\ \text{LSI}_{PN} \text{ - changes of labour demand by occupations and sectors;} \\ \text{N - sector.} \end{array}$ 

Formulas 26 - 28 create the transition from the sub-model of occupational labour demand to sub-model of labour demand by education. These formulas have a support function in the model, but it is not practicable to create a special sub-model.

The sub-model of labour demand by education has only one stock - "labour demand by fields, levels of education and occupations". The calculation equation is presented in formula 29:

$$LS_{JLP}(t) = LS_{JLP}(t_0) + \int_{t_0}^{T} (LSP_{JLP} - LSS_{JLP}) dt, \qquad (29)$$

where

 $LS_{JLP}$  - labour demand by fields, levels of education and occupations;  $LSP_{JLP}$  - growth of labour demand by fields, levels of education and occupations;  $LSS_{JLP}$  - decrease of labour demand by fields, levels of education and occupations.

The initial level of stock is determined on the basis of statistical data, by the means of Latvian Occupational Classification (3-digit level of occupation code) of 127 occupational group units, aggregation of levels of education of 5 units and 79 fields of education (on the basis of the classification of the Ministry of Education and Science of the Republic of Latvia, third, fourth and fifth groups). Stock changes are determined by the flows "growth of labour demand by fields, levels of education and occupations" and "decrease of labour demand by fields, levels of education and occupations" that divide the changes of labour demand in occupations by levels and fields of education. This calculation equations are presented in formulas 30 and 31:

$$LSS_{JLP_t} = -LSS_{P_t} \times LSSt_{JLP_t}, \tag{30}$$

$$LSP_{JLPt} = NLSPA_{JLPt} + LSPL_{JLPt} + LSPM_{JLPt}, \qquad (31)$$

where

LSS<sub>JLP</sub> - decrease of labour demand by fields, levels of education and occupations;

LSS<sub>P</sub> - decrease of labour demand by occupations;

LSSt<sub>JLP</sub> - structure of labour demand by fields, levels of education and occupations;

LSP<sub>JLP</sub> - growth of labour demand by fields, levels of education and occupations;

LSPA<sub>JLP</sub> - growth of labour demand by fields, levels of education and occupations in accordance with the compliance of education to occupation;

 $LSPL_{JLP}$  - growth of labour demand by fields, levels of education and occupations in accordance with the structure of labour demand by educational levels;

 $LSPM_{JLP}$  - growth of labour demand by fields, levels of education and occupations in accordance with the structure of educational compliance matrix.

As seen in formulas 30 and 31, changes of labour demand by fields, levels of education of education and occupations are also divided into two parts: growth and reduction. Division of changes into two parts is associated with the application of different algorithms in every part. Calculation of reduction is simpler, but the growth has the more difficult calculation and consists of three elements.

At first, the algorithm is being analysed in details and this algorithm is used in the calculations of reduction of labour demand by fields, levels of education and occupations. Formula 30 shows that it is related to the reduction in labour demand by occupations and structure of labour demand by fields, levels of education and occupations.

The general scheme of the calculation algorithm of reduction of labour demand by fields, levels of education and occupations is presented in Figure 2.13.



Figure 2.13. The general scheme of the calculation algorithm of reduction of labour demand by fields, levels of education and occupations

Figure 2.13 shows that the decrease of labour demand by occupations according to the structure of labour demand by fields, levels of education and occupations is divided into fields, levels of education and occupations. As a result, it leads to changes of labour demand in the mentioned dimensions. The economic essence of this algorithm is simple: by reducing the labour demand in occupation, the labour demand in occupation by education groups decreases in proportion to labour demand in education groups. The reduction of labour demand by occupations is analysed in formula 26, but the calculation of the structure of labour demand by fields, levels of education and occupations is presented in formula 32:

$$LSSt_{JLPt} = \begin{cases} \frac{LS_{JLPt}}{\sum \sum_{i \in J} LS_{JLPt}^{ki}}, \sum_{k \in L} \sum_{i \in J} LS_{JLPt}^{ki} > 0\\ 0, \sum_{k \in L} \sum_{i \in J} LS_{JLPt}^{ki} \le 0 \end{cases},$$
(32)

where

LSSt<sub>JLP</sub> - structure of labour demand by fields, levels of education and occupations; LS<sub>JLP</sub> - labour demand by fields, levels of education and occupations; L - level of education; J - field of education;

Formula 32 presents that the structure of labour demand by fields, levels of education and occupations is being calculated by dividing the labour demand by fields, levels of education and occupations (matrix) by the total labour demand in the occupation by fields and levels of education (vector).

Additionally, formula 32 shows that under specific conditions (when there is not employed any person in the occupation and /or there is no labour demand in the occupation) it is not mathematically possible to calculate the structure of labour demand. Under these conditions, formula 32 assigns 0 value (no structure) for all elements related to occupations.

The growth of labour demand by fields, levels of education and occupations (formula 31) is divided into three parts:

1) in accordance with the education-occupational compliance, including the ratio of fields and levels of education;

2) in accordance with the education-occupational compliance, including the ratio of levels of education;

3) in accordance with the education-occupational compliance.

Each of these three algorithms divides the growth of labour demand in occupation by fields and levels of education, and is forming a single three-step algorithm. The general scheme of algorithm calculation of the growth of labour demand by fields, levels of education and occupations is presented in the Figure 2.14.



Figure 2.14 The general scheme of algorithm calculation of the growth of labour demand by fields, levels of education and occupation

With the growth of labour demand in occupations, only the specialists with the appropriate education should be employed, taking into account the level and field of education, which is the first priority of the model (or division of the growth of labour demand by fields and levels of education at the first stage of the algorithm). It is not always possible to employ the specialists with appropriate education. In this case, the growth of labour demand in occupations, which were not previously divided by educational groups, is being divided in accordance with the structure of levels of education (only those specialists are being employed, whose level of education meets the occupational requirements), which the second priority of the model. Theoretically, there is one more algorithm of growth of labour demand by education groups and occupations: when the new occupations are being created, or the occupation has new educational requirements and / or no employee does not correspond to the occupation. In this case there may not be occupational and educational structure, and employees should be recruited directly in accordance with the requirements for their education. This is the third model priority, which analyses only the undivided growth of labour demand in the first two priorities. Each of these algorithms of growth of labour demand will be analysed separately - the first is the growth of labour demand by the levels of education and occupations, taking into account the compliance of education to the requirements of the occupation, whose calculation equation is presented in formula 33:

$$LSPA_{JLPt} = LSStA_{JLPt} \times LSP_{Pt}, \qquad (33)$$

where

 $LSPA_{JLP}$  - growth of labour demand by fields, levels of education of education and occupations, in accordance with the compliance of education to requirements of occupation;

 $LSStA_{JLP}$  - structure of labour demand in the occupation with the appropriate education by fields and levels of education;

LSP<sub>P</sub> - growth of labour demand by occupations.

Formula 33 shows that the growth of labour demand by occupations is divided into fields, levels of education in accordance with the structure of labour demand in the occupation with the appropriate education. The compliance of the occupations and education is determined by the

educational compliance matrix. Calculation of the structure of labour demand with the appropriate education is presented in formula 34:

$$LSStA_{JLPt} = \begin{cases} \frac{LSA_{JLPt}}{\sum \sum_{k \in L} \sum_{i \in J} DSPA_{JLPt}^{ki}}, \sum_{k \in L} \sum_{i \in J} LSA_{JLPt}^{ki} > 0\\ 0, \sum_{k \in L} \sum_{i \in J} LSA_{JLPt}^{ki} \le 0 \end{cases},$$
(34)

where

 $LSStA_{JLP}$  - structure of labour demand in the occupation with the appropriate education by fields and levels of education;

 $LSA_{JLP}$  - labour demand in the occupation with the appropriate education by fields and levels of education.

Structure of labour demand in the occupation with the appropriate division of education by fields and levels of education is being calculated by dividing the labour demand in the occupation by the appropriate education by fields and levels of education (matrix) with a total labour demand in the occupation with the appropriate education (vector).

Additionally, formula 34 shows that under specific conditions (when the occupation does not employ any person and/or there is no labour demand in the occupation) it is not mathematically possible to calculate the structure of labour demand. Under these conditions, formula 34 assigns 0 value (no structure) for all elements related to occupations.

Calculation of labour demand in occupation with the appropriate education by fields and levels of education is presented in formula 35:

$$LSA_{JLPt} = LS_{JLPt} \times IAM , \qquad (35)$$

where

 $LSA_{JLP}$  - labour demand in the occupation with the appropriate education by fields and levels of education;

 $LS_{JLP}$  - labour demand by fields, levels of education and occupations;

IAM - educational compliance matrix.

Labour demand in the occupation with the appropriate education by fields and levels of education is being calculated by multiplying the labour demand by fields, levels of education and occupations by educational compliance matrix. In this way, the labour demand is being selected from the total demand, which refers to the occupations with the appropriate education. Beforehand it is presented (formula 34) how their structure, according to which the growth of labour demand is divided into educational fields and levels (formula 33), is calculated.

Theoretically, this algorithm should fully support the growth of labour demand in occupations by fields and levels of education. In practice, the quality of the educational compliance matrix as well as the quality of statistical data has a huge impact. If the educational compliance matrix determines that a specific education is necessary for the occupational performance, but statistics shows that there are no employed persons with the appropriate education in the occupation, the algorithm will never ensure the growth of labour demand in the analysed occupations with the appropriate education. In order to close this, the algorithm of the growth of labour demand by fields and levels of education is complemented with two other parts (sub-algorithms). It is described in more detail below.

As stated above, the sub-algorithm does not fully ensure the division of occupational labour demand by fields and levels of education. The division level is being calculated in the model (formula 36):

$$LSPA_{Pt} = \sum_{k \in L} \sum_{i \in J} LSPA_{JLPt}^{ki}, \qquad (36)$$

where

 $LSPA_P$  - growth of labour demand by occupations, in accordance with the compliance of education to the requirements of occupation;

 $LSPA_{JLP}$  - growth of labour demand by fields, levels of education of education and occupations, in accordance with the compliance of education to occupation;

L - level of education;

J - field of education.

In fact, formula 36 presents the return path: the growth of labour demand by fields, levels of education and occupations has the reduced aggregation to occupational level. It allows analysing the divided labour demand in terms of occupation by fields and levels of education, as well as to compare it to the growth of labour demand by occupations, which are presented in formula 37:

$$LSPnA_{p_t} = LSP_{p_t} - LSPA_{p_t}, \qquad (37)$$

where

 $LSPnA_P$  - growth of labour demand by occupations, without regard to the compliance of education to the requirements of occupation;

LSP<sub>P</sub> - growth of labour demand by occupations;

 $LSPA_P$  - growth of labour demand by occupations, in accordance with the compliance of education to the requirements of occupation.

By subtracting the divided growth of labour demand in terms of occupation from the growth of labour demand by occupations, the "undivided balance" is being calculated, which needs to be divided later. In the model it is marked as "growth of labour demand by occupations, without regard to compliance of education to the requirements of occupation".

The second stage sub-algorithm of the growth of labour demand by occupations is related to the structure of labour demand levels of education. This sub-algorithm divided previously undivided growth of labour demand by educational levels. This algorithm is simple, and the essence is the following: in accordance with the structure of labour demand by educational levels, previously undivided growth of labour demand by occupations and structure of educational compliance matrix the growth of labour demand by fields, levels of education and occupations is being calculated (formula 38):

$$LSPL_{JI,Pt} = LSPnA_{Pt} \times LSSt_{I,Pt} \times IAMSL_{JI,Pt}, \qquad (38)$$

where

 $LSPL_{JLP}$  - growth of labour demand by fields, levels of education and occupations, in accordance with the structure of labour demand by levels of education;

 $LSPnA_P$  - growth of labour demand by occupations, without regard to the compliance of education to the occupation;

LSSt<sub>LP</sub> - structure of labour demand by levels of education and occupations;

IAMSL<sub>JLP</sub> - structure of educational compliance matrix by levels of education.

The elements, presented in formula 38, define only the growth of labour demand by occupations, without regard to the compliance of the education to the occupation. Structure of labour demand by levels of education and occupations is calculated in formula 39, but the calculation of the structure of educational compliance matrix is presented in formula 40:

$$LSSt_{LPt} = \frac{\sum_{i \in J} LS_{JLPt}^{i}}{\sum_{k \in J} \sum_{i \in L} \sum_{n \in P} LS_{JLPt}^{kin}},$$
(39)

where

 $LSSt_{LP}$  - structure of labour demand by levels of education and occupations;  $LS_{ILP}$  - labour demand by fields, levels of education and occupations;

L - level of education;

J - field of education;

P - occupation.

$$IAMSL_{JLPt} = \begin{cases} \frac{IAM}{\sum_{k \in J} IAM^{k}}, \sum_{k \in J} IAM^{k} > 0\\ 0, \sum_{k \in J} IAM^{k} \le 0 \end{cases},$$
(40)

where

IAMSL<sub>JLP</sub> - structure of educational compliance matrix by levels of education; IAM - educational compliance matrix; J - field of education.

At the second stage the observation of three components in formula 38 divides the demand on occupations of labour by educational levels in accordance with existing structure of labour demand educational levels, and further, without the regard to the current labour demand and its structure, according to the educational compliance matrix structure, divides into fields of education. This action allows reducing the effect of the lack of statistical data and educational compliance matrix, and changes the structure of labour demand under specific circumstances, for example, when the new educational requirements arise.

However, the second stage sub-algorithm does not ensure an adequate functioning of the model for new occupations or occupations that have not previously employed any people. The growth of labour demand by occupations will not be divided by fields and levels of education. In order to avoid the possible problem, the division of the growth of occupational labour demand at the third stage is being offered. But before the examination it is necessary to determine the level of labour demand, which is not divided into occupations and education groups at the first two stages. Second stage level is calculated in a similar way to first stage level, i.e., by determining the divided volume in the sub-algorithm, and calculating the difference between divided and undivided labour demand (formulas 41 and 42):

$$LSPL_{Pt} = \sum_{k \in L} \sum_{i \in J} LSPL_{JLPt}^{ki} , \qquad (41)$$

$$LSPnAL_{Pt} = LSPnA_{Pt} - LSPL_{Pt}, \qquad (42)$$

where

 $LSPL_P$  - growth of labour demand by occupations, in accordance with the structure of labour demand by levels of education;

 $LSPL_{JLP}$  - growth of labour demand by fields, levels of education and occupations, in accordance with the structure of labour demand by levels of education;

L - level of education;

J - field of education;

LSPnAL<sub>P</sub> - growth of labour demand by occupations, without regard to the compliance of education to the requirements of occupation and structure of labour demand by levels of education;

 $LSPnA_P$  - growth of labour demand by occupations, without regard to the compliance of education to the requirements of occupation.

The calculated growth of labour demand by occupations, without regard to the compliance with the requirements of the occupation and structure of labour demand by levels of education, further, at the third stage, is divided in direct ratio to educational compliance matrix. This means that if the occupation required the single-field and single-level education, the entire growth of labour demand in the occupation causes the demand only in a single education group, if the education is required in two groups, this causes the demand in two education groups in the same proportion, etc.

This sub-algorithm defines the structure of educational matrix (formula 43) and in accordance to this, the growth of labour demand by fields, levels of education and occupations, in accordance with the structure of educational compliance matrix, is being calculated (formula 44):

$$IAMSM_{JLP} = \begin{cases} \frac{IAM}{\sum \sum_{i \in J} IAM^{ki}}, \sum_{k \in L} \sum_{i \in J} IAM^{ki} > 0\\ 0, \sum_{k \in L} \sum_{i \in J} IAM^{ki} \le 0 \end{cases},$$
(43)

$$LSPM_{JLP_{t}} = LSPnAL_{P_{t}} \times IAMSM_{JLP}, \qquad (44)$$

where

IAMSM<sub>JLP</sub> - structure of educational compliance matrix;

IAM - educational compliance matrix;

 $LSPM_{JLP}$  - growth of labour demand by fields, levels of education and occupations, in accordance with the structure of the educational compliance matrix;

 $LSPnAL_P$  - growth of labour demand by occupations, without regard to the compliance of education to the requirements of occupation and structure of labour demand by levels of education.

The analysed three-step algorithm allows adequate division of the growth of labour demand by fields and levels of education.

#### 2.1.4. Modelling labour demand by gender

The sub-model of labour demand by gender is being based on the sub-model of the labour demand by occupations, as well statistical data on gender and occupations, and determines the labour demand by gender.
In order to ensure the operation of the sub-model at the forecasting time, the sub-model of sectorial labour demand, sub-model of occupational labour demand (with all those incoming indexes and data), as well as statistical data on the number of employees by occupations and gender in the base year should be available.

Some elements are taken from the sub-model of labour demand by education, but they are calculated in the sub-model of labour demand by education on the basis of the other sub-models. They have input data functions in the sub-model of labour demand by education; the given sub-model has no effect on them. They can be calculated in other sub-models, so that would indicate that the sub-model of labour demand by gender technically (functionally) is related to the sub-model of labour demand by education. But there is no full functional relationship between these two sub-models.

The sub-model of labour demand by gender has only one stock - "labour demand by fields, levels of education, occupations and gender". The calculation equation is presented in formula 45:

$$LS_{JLDP}(t) = LS_{JLDP}(t_{0}) + \int_{t_{0}}^{T} (LSP_{JLDP} - LSS_{JLDP}) dt , \qquad (45)$$

where

 $LS_{JLDP}$  - labour demand by fields, levels of education, occupations and gender;  $LSP_{JLDP}$  - growth of labour demand by fields, levels of education, occupations and gender;  $LSS_{JLDP}$  - decrease of labour demand by fields, levels of education, occupations and gender;

Stock level in the base period is determined from statistical data. Stock changes determine the flow of "growth of labour demand by occupations and gender" and "decrease of labour demand by occupations and gender". These calculation equations are presented in formulas 46 and 47:

$$LSP_{JLDPt} = LSP_{JLPt} \times LSSt_{DPt}, \qquad (46)$$

$$LSS_{JLDPt} = LSS_{JLPt} \times LSSt_{DPt}, \qquad (47)$$

where

 $LSP_{JLDP}$  - growth of labour demand by fields, levels of education, occupations and gender;  $LSS_{JLDP}$  - decrease of labour demand by fields, levels of education, occupations and gender;  $LSP_{JLP}$  - growth of labour demand by fields, levels of education and occupations;  $LSS_{JLP}$  - decrease of labour demand by fields, levels of education and occupations;  $LSS_{JLP}$  - structure of labour demand by occupations and gender.

By analysing the equations of changes of labour demand by occupations and gender, it is evident that the changes by gender are associated with changes in the occupations and observe the structure of labour demand by occupations and gender. The similar equations are used for the growth and decrease of labour demand by occupations and gender, as well as a single algorithm is being used. This is done in order to facilitate the further model development: the further perspective, by combining the sub-models of gender supply and demand, decrease of the number of employees by gender would be associated with the decrease of gender demand, but the growth would be corrected in accordance with the supply structure.

The calculation equation of the structures of labour demand by occupations and gender are presented in formula 48:

$$LSSt_{DPt} = \begin{cases} \sum_{k=J} \sum_{n=L} LS_{JLDPt}^{kn}, \sum_{k=J} \sum_{n=L} \sum_{i \in D} LS_{JLDPt}^{kni} > 0\\ \sum_{k=J} \sum_{n=L} \sum_{i \in D} LS_{JLDPt}^{kni} \le 0 \end{cases},$$
(48)

 $LSSt_{DP}$  - structure of labour demand by the occupations and gender;  $LS_{JLDP}$  - labour demand by fields, levels of education, occupations and gender; D - gender.

Structure of labour demand by occupations and is calculated by dividing the labour demand by occupations and gender (matrix) with the labour demand by gender (vector).

Additionally, formula 47 shows that under specific conditions (when the occupation does not employ any person and/or there is no labour demand in the occupation) it is not mathematically possible to calculate the structure of labour demand. Under these conditions, formula 47 assigns 0 value (no structure) for all elements related to occupations.

In order to ensure the observance of the requirements of MoE for the obtained results, labour demand by occupations and gender has the reduced the number of dimensions, by summing up the labour demand for gender by occupations, formula 49:

$$LS_{Dt} = \sum_{k=J} \sum_{n=L} \sum_{i\in P} LS_{JLDPt}^{kni}, \qquad (49)$$

where  $LS_D$  - labour demand by gender;  $LS_{DP}$  - labour demand by occupations and gender; P - occupations.

The calculation of the labour demand by gender in the sub-model has largely a technical function; this index is not being used in the sub-model.

## 2.2. Labour supply module

Labour supply module simulates the demographic processes, division of the population by ages, gender, economic activity, education and occupation. Labour supply module consists of submodels of demography, education attainment, working-age population stock-taking, labour force participation and international migration. Module structure is being presented in the Figure 2.15.



Labour supply module of SDM model



In the demographic sub-model of labour supply module the population is defined by age for one-year groups and gender. This sub-model defines the birth rate, mortality and aging of population. By reaching the age of seven years, the demographic sub-model defines the incoming population of education system. By reaching working age, the demographic sub-model defines the labour growth (in working-age population stock-taking sub-model), in accordance with prior education (education attainment sub-model) and the anticipated economic activity (labour force participation sub-model). By reaching retirement age, demographic sub-model defines the labour decrease. The same happens in case of death before retirement age.

Working-age population stock-taking sub-model represents the labour structure by 5-year age groups, gender, economic activity, education and occupations, that is, represent the operating results of other sub-models.

Labour force participation sub-model defines the labour structure in the field of economic activity.

Education attainment sub-model defines not only the increase of the primary labour volume (along with demographic sub-model), but also the changes of the labour along with education attainment, including lifelong education.

International migration sub-model defines the change of population and labour along with international migration processes.

The sub-models of labour demand module are examined in the following subsections.

### 2.2.1. Demographic sub-model

For the demographic modelling in SDM model the classical scheme is being applied: the central place belongs to population stock; it can be complemented by incoming flows (e.g., birth

rate), devastated by outgoing flows (such as mortality), as well as the age structure can be changed by the means of aging algorithm.

Before the identification of the SDM demographic sub-model structure, the connection with other sub-models is being examined.

Birth rate should be dependent on the economic situation in the country, i.e. GDP per capita. For this purpose, the new element of "GDP per capita" is being created in the model. In order to make the model more user friendly, this element is calculated automatically from the existing model parameters.

The elements related to GDP per capita and the calculation of its changes is presented in Figure 2.16.



Figure 2.16. GDP per capita and the calculation of its changes

Figure 2.16 represents that the algorithm consists of three elements: GDP per capita, the established GDP per capita in the previous period, and the GDP growth rate. Total GDP per capita is being calculated by summing up GDP by sectors and dividing it by the total population; its calculation equation is presented in formula 50:

$$GDPI_{t} = \frac{\sum_{k=N} GDP_{Nt}^{k}}{\sum_{i \in V} \sum_{j \in D} I_{DVt}^{ij}},$$
(50)

where GDPI - total GDP per capita; GDP<sub>N</sub> - GDP forecasts by sectors;  $I_{VD}$  - population; V - age; D - gender.

For calculation of total GDP per capita the used GDP is derived from the labour demand module and has been considered above, but the population will be analysed further.

Total GDP per capita is being delayed by one year, thus, calculating the total GDP per capita in the previous period (see formula 51):

$$IPIKPI_{t} = IKPI_{t-1}, (51)$$

where IPIKPI - total GDP per capita in the previous period; GDPI - total GDP per capita.

It is possible to calculate the GDP growth rate from the total GDP per capita and total GDP per capita in the previous period (see formula 52):

$$IKPPT_{t} = \frac{GDPI_{t}}{IPIKPI_{t}},$$
(52)

where IKPPT - growth rate of total GDP per capita; IPIKPI - total GDP per capita in the previous period; GDPI - total GDP per capita.

The GDP calculation, as well as identification of changes in the demographic sub-model has a technical role, but its elements are used in a variety of sub-model algorithms.

The algorithm for calculation and changes of population is presented in the Figure 2.17.



Figure 2.17. Calculation of population

Figure 2.17 presents the calculation logic of the population. Population repository has been changed (complemented or reduced) by the respective flows: birth rate, mortality, aging and migration.

The main stock of the SDM model demographic sub-model (population stock) is being characterized by the equation (formula 53):

$$I_{VD}(t) = I_{VD}(t_0) + \int_{t_0}^{T} (D_D - M_{VD} - MI_{VD} - N_{(V-1)D} + N_{VD}) dt,$$
  

$$V = \begin{cases} \{1, 2, \dots, 99, 100 + \}, V \in N_{(V-1)D} \lor N_{VD} \\ \{0, 1, \dots, 99, 100 + \}, V \in I_{VD} \lor M_{VD} \lor MI_{VD} \end{cases},$$
(53)

where  $I_{VD}$  - population;  $D_D$  - birth rate;  $M_{VD}$  - mortality;  $MI_{VD}$  - migration;  $N_{VD}$  - aging; V - age.

By analysing formula 53, it is evident that the population is dependent on the initial level and changes, consisting of birth rate, mortality, migration and aging. Aging is increasing the population of certain ages and at the same time reducing the population of younger ages. The aging dimension is being reduced by one group (0 years old), as it not possible to calculate the population that is one year younger than this age.

Aging is the only element in formula 53, which calculation does not involve a special algorithm; its calculation is simple and is described in one equation. Therefore, the further explanation of the model is being started from this element. Aging calculation equation is presented in formula 54:

$$N_{\rm VDt} = I_{\rm (V-1)D(t-1)} - M_{\rm (V-1)D(t-1)} - MI_{\rm (V-1)D(t-1)},$$
  

$$V = \{1, 2, \dots, 99, 100 + \},$$
(54)

where  $N_{VD}$  - aging;  $I_{VD}$  - population;  $M_{VD}$  - mortality;  $MI_{VD}$  - migration; V - age.

By analysing the 54 equation, it is visible that the change of population (aging) in the current period, for the relevant age depends on the previous period's population that is one year younger, and from the mortality and migration of this group. Aging determines decrease of population of 0 age, and both decrease and increase of other ages.

Formula 55 presents the birth rate flow:

$$D_{Dt} = \sum_{i \in V} D_{DVt}^{i} , \qquad (55)$$

where  $D_D$  - birth rate;  $D_{DV}$  - the number of born people by gender and maternal age; V - age.

Birth rate equation sums up the number of born people by maternal age, preserving a separation by gender. The number of new-borns by gender and maternal age is the result of the birth rate calculation algorithm, which will be considered further.

Formula 56 presents the mortality flow:

$$M_{VDt} = I_{VDt} \times MK_{VDt} , \qquad (56)$$

where  $M_{VD}$  - mortality;  $I_{VD}$  - population;  $MK_{VD}$  - death rates.

The mortality is being calculated by multiplying the mortality index for one-year age groups and by gender by a population in the corresponding groups. Mortality index reflects the result of the mortality index calculation algorithm, which will be considered further.

Calculation of migration is presented in formula 57:

$$MI_{VDt} = EM_{VDt} - IM_{VDt}, \qquad (57)$$

where  $MI_{VD}$  - migration;  $EM_{VD}$  - emigration;  $IM_{VD}$  - immigration.

Migration flow represents the balance of emigration and immigration.

Then the flow-forming algorithms will be analysed, the first of which is related to birth rate. Birth rate algorithm is a complex, which can be explained in two stages: changes of birth rate index in accordance with the changes in GDP and changes of birth rate in accordance with birth rate index and demographic changes. It is visually represented in Figure 18.2.



Figure 2.18. Birth rate calculation algorithm

The description of birth rate calculation algorithm starts from the change of birth rate index algorithm.

Considering GDP per capita, there are two calculation sub-algorithms of birth rate index: one - for reduction of GDP per capita, and the other - for growth of GDP per capita. Algorithms reflect GDP impact on the birth rate indexes: by growing GDP increases birth rate index, by reducing GDP, decreases birth rate index. Decrease of birth rate index in case of reducing GDP is being characterized by formula 58:

$$DKSSA_t = DKS_t \times IKPPT_t, \qquad (58)$$

where

DKSSA - total birth rate per woman of reproductive age in case of GDP reduction; IKPPT - total growth rate of GDP per capita; DKS - birth rate per woman of reproductive age.

As we can see in formula 58, the total birth rate per woman of reproductive age in case of GDP reduction (i.e., birth rate index) is directly dependent on changes of GDP per capita. The decrease of GDP per capita causes the same decrease of the birth rate index. This model equation has been composed on the basis of available statistics of decrease of GDP. The next formula 59 analyses the birth rate index in case of GDP growth:

$$DKSPA_{t} = DKND + DKRK \times IPIKPI_{t}, \qquad (59)$$

where

DKSPA - total birth rate per woman of reproductive age in case of GDP growth;

DKND - independent part of birth rate index;

DKRK - regression index of birth rate index;

IPIKPI - total GDP per capita in the previous period.

Formula 59 shows that the birth rate index in case of GDP (per capita) grows linearly depend on GDP per capita in the previous period. One-year delay reflects that the birth rate is not growing in the first year when there is the growth of well-being and GDP, but in the following year. This equation and its form have been developed in accordance with the requirements of MoE and statistical correlations between birth rate per woman of reproductive age and GDP per capita in the previous period.

However, formula 59 may show inadequate results of the post-crisis period, when GDP may be renewed very quickly (i.e., with a high growth rate), while the birth rate is not growing rapidly. To avoid this problem, the model involves the constraining sub-algorithm of growth rate of birth rate index, which is schematically shown in Figure 2.19.



Figure 2.19. Constraining sub-algorithm of growth rate of birth rate index

The essence of the constraining sub-algorithm of growth rate of birth rate index is the following: from the birth rate index and birth rate per woman of reproductive age, calculated in formula 59 (from the related sub-algorithm, this index will be analysed further) is calculated the total growth rate of birth rate index. The calculated growth rate is compared with a growth rate of birth rate index in the long-term period, and the calculated birth rate index is adjusted accordingly. Growth rate of birth rate index in the long-term period is not calculated in this sub-algorithm and will be considered further. Formula 60 presents the calculation of the total growth rate of birth rate index:

$$DKPT_{t} = \frac{DKSPA_{t}}{DKS_{t-1}},$$
(60)

where DKPT - total growth rate of birth rate; DKSPA - total birth rate per woman of reproductive age in case of GDP growth; DKS - birth rate per woman of reproductive age.

Formula 60 shows that the growth rate is calculated by dividing the total birth rate per woman of reproductive age in case of GDP growth by the birth rate per woman of reproductive age in the previous period (from the related sub-algorithm). In this case, the total birth rate per woman of reproductive age in case of GDP growth defines only the indicative growth of birth rate, but it is not possible always to realize, taking into account the internal constraints of the model. Therefore, the growth rate calculation is based on the actual level which reflects the birth rate per woman of reproductive age, rather than indicative level. Formula 61 presents the comparison of growth rate and adjustment of birth rate index:

$$DKPTiIPT_{t} = \begin{cases} DKSPA_{t}, DKPTIP_{t} \ge DKPT_{t} \\ \left(\frac{DKPTIP_{t} + DKPT_{t}}{2} \times DKS_{t}\right) \land DKSPA_{t}, DKPTIP_{t} < DKPT_{t}, \end{cases}$$
(61)

DKPTiIPT - limitation of total birth rate index growth in accordance with the long-time growth rate;

DKSPA - total birth rate per woman of reproductive age in case of GDP growth; DKPTIP - growth rate of birth rate in the long-term period; DKPT - total growth rate of birth rate; DKS - birth rate per woman of reproductive age.

Formula 61 shows that, if the growth rate of the birth rate index in the long-term period is greater than in the current period, the birth rate index is not being changed, however, if the growth rate of the birth rate in the current period is greater than the changes in the long-term period, the birth rate is adjusted. The average growth rate in the current and long-term period are being calculated for the adjustment, and the average annual growth rate is being multiplied by the birth rate per woman of reproductive age (i.e., the birth rate index), the resulting element being compared to the index "total birth rate per woman of reproductive age in case of GDP growth" and the minimal index is being selected to prevent the exceed of the constrained index over the limitable index. In the result this equation provides a gradual increase of birth rate without the "acceleration" after of GDP crises.

The regression equation, applied in formula 59, has the disadvantage that it has a linear form. However, having examined the statistical data, this form reflects the reality of the last decade better than other forms. The linear regression equation indicates an infinite growth in case of infinite GDP growth, but this does not correspond to statistics or theoretical assumptions. Upon achievement of certain level, GDP growth does not cause the growth of birth rate. So the constraint should be introduced into the equation, which causes the stop of the growth of birth rate. This constraint is being analysed in formula 62:

$$DKPTiMR_{t} = \begin{cases} DKPTiIPT_{t}, DKSM_{t} \ge DKPTiIPT_{t} \\ DKSM_{t}, DKSM_{t} < DKPTiIPT_{t} \end{cases},$$
(62)

where

DKPTiMR - limitation of total birth rate index growth in accordance with the maximum threshold;

DKPTiIPT - limitation of total birth rate index growth in accordance with the long-time growth rate;

DKSM - maximum total birth rate per woman.

Formula 62 shows that until the birth rate is less than the set constraint, it is not being adjusted, but above this constraint the birth rate is being replaced by the maximum birth rate. The maximum birth rate constraint in the model is set at the average EU level.

So, it was investigated the calculation of the birth rate index in case of GDP reduction (formula 58) and GDP growth (formulas 59 - 62). The next equation (formula 63) presents the choice of index, depending on the changes of GDP:

$$KDKI_{t} = \begin{cases} DKPTiMR_{t}, IKPPT_{t} \ge 1\\ DKSSA_{t}, IKPPT_{t} < 1 \end{cases},$$
(63)

KDKI - the choice of total birth rate index;

DKPTiMR - limitation of total birth rate index growth in accordance with the maximum threshold;

IKPPT - the growth rate of total GDP per capita;

DKSSA - total birth rate per woman of reproductive age in case of GDP reduction.

Formula 63 reflects that the choice of the total birth rate index will depend on the GDP growth rate; if it is greater than one, the algorithm of birth rate index calculation in case of GDP growth is being applied, but if it is less than one, the algorithm of birth rate index calculation in case of GDP reduction is being applied.

The impact of change of GDP on birth rates index is in general described in Figure 2.20.



Figure 2.20. Algorithm of changes of birth rate index

As described above, calculation of the birth rate is different in case of GDP growth or reduction. In case of GDP reduction, it is a simple and direct connection with changes of GDP. In case of GDP growth it is being defined by the regression equation and adjusted in accordance with the growth rate in a long-term period and birth rate potential maximum.

It is important to note that the above mentioned algorithm of calculation and choice of birth rate index describes the total change of the birth rate from the total GDP. Then it is necessary to divide the total birth rate per woman by women ages. The model is based on the assumption that GDP growth in the same ratio affects birth rate growth by women age. This is being analysed by the sub-algorithm of the change of birth rate, which is based on the change of birth rate index as well as the population. First, it describes the connection of population with birth rate and birth rate indexes (see Figure 2.21).



Figure 2.21. Connection of birth rate, birth rate index and population

Population and birth rates have been analysed previously (in formulas 53 and 55, Figures 2.17 and 2.18). The main thing, reflected by Figure 2.21 is that the women are being selected, while maintaining the age structure and the selected number is being multiplied by the birth rate, thus obtaining the number of born people by gender and maternal age.

Calculation of the number of women is presented in formula 64, but the number of born people by gender and maternal age - in formula 65:

$$S_{Vt} = I_{VDt}, D \in S, \tag{64}$$

where  $S_V$  - number of women;  $I_{VD}$  - population; D - gender; S - female gender.

$$D_{DVt} = DK_{DSVt} \times S_{Vt}, \tag{65}$$

where  $D_{DV}$  - number of born people by gender and maternal age;  $S_V$  - number of women;  $DK_{DSV}$  - birth rate index.

In the previous equations (formulas 58 - 63) the total birth rate (per woman of reproductive age) has been analysed. Formula 65 shows the birth rate indexes by gender and maternal age. In order to combine the total birth rate index (scalar) and birth rate (matrix which reflects indexes by gender and maternal age), the model involves sub-algorithm of changes of birth rate, presented in Figure 2.22.



Figure 2.22. Algorithm of changes of birth rate index

Figure 2.22 presents that the women of reproductive age have been selected from the total number of women. The birth rate per woman of reproductive age (ie overall index, scalar value) is being calculated form the number of born people by gender and maternal age. This index represents the total birth rate, taking into account the population. This can be compared to the "choice of total birth rate index", which shows, which should be the birth rate index, considering the changes of GDP. As the birth rate per woman of reproductive age is being expressed in the birth rate, it can be called as the total actual birth rate.

By dividing the index of "choice of total birth rate" by birth rate per woman of reproductive age (or actual total birth rate index), it is possible to determine which changes should take place in the birth rate in accordance with the changes of GDP and population. By multiplying the expected changes of the birth rate, the "calculated birth rate indexes" are being calculated, i.e., which should be the birth rate indexes in accordance with the changes of GDP and population.

Birth rates indexes are being changed in accordance with the difference between the birth rate indexes and calculated birth rate indexes, i.e. the change of birth rate indexes.

The main part in this algorithm belongs to the birth rate stock, which is presented in formula 66:

$$DK_{\rm DSV}(t) = DK_{\rm DSV}(t_0) + \int_{t_0}^{T} DKI_{DSV} dt$$
, (66)

where  $DK_{DSV}$  - birth rate indexes;  $DKI_{DSV}$  - change of birth rate indexes.

Calculation of the change of birth rate indexes is presented in formula 67:

$$DKI_{\rm DSVt} = DK_{\rm DSVt} - ADK_{\rm DSVt}, \qquad (67)$$

where  $DKI_{DSV}$  - change of birth rate indexes;  $DK_{DSV}$  - birth rate indexes;  $ADK_{DSV}$  - calculated birth rate indexes.

Determination of the calculated birth rate indexes is presented in formula 68:

$$ADK_{\rm DSVt} = DK_{\rm DSVt} \times \frac{KDKI_t}{DKS_t},$$
(68)

where  $ADK_{DSV}$  - calculated birth rate indexes;  $DK_{DSV}$  - birth rate indexes; KDKI - choice of the total birth rate index; DKS - birth rate per woman of reproductive age.

Calculation of birth rate per woman of reproductive age is presented in formula 69, but the calculation of the number of women of reproductive age is presented in formula 70:

$$DKS_{t} = \frac{\sum_{i \in V} \sum_{j \in D} D_{DVt}^{ij}}{\sum_{i \in V} SR_{Vt}^{i}},$$
(69)

where

DKS - birth rate per woman of reproductive age;

SR<sub>V</sub> - number of women of reproductive age;

D<sub>DV</sub> - number of born people by gender and maternal age;

D - gender;

V - age.

$$SR_{\rm Vt} = S_{\rm Vt}, V \in \{15, 16, \dots, 44, 45\},\tag{70}$$

where

 $SR_V$  - number of women of reproductive age;  $S_V$  - number of women; V - age.

By explaining the birth rate algorithm, another index should be considered - the growth rate of birth rate in the long-term period, which has been used in formula 60. The growth rate of birth rate in the long-term period has been analysed in formula 71:

$$DKPTIP_{t} = \frac{\frac{DKS_{t} - DKS_{t-i}}{i}}{DKS_{t-i}} + 1,$$
(71)

where

DKPTIP - growth rate of birth rate in the long-term period; DKS - birth rate per woman of reproductive age; i -birth rate distribution time.

Formula 71 shows that at first the difference between the birth rate in the beginning and in the end of the period is being determined, then the difference is being divided by the number of years in the long-term period, obtaining the average annual changes; the average annual changes are divided by the initial birth rate index, i.e., obtaining relative index. This sub-section will be continued with the mortality algorithm interpretation. Figure 2.23 represents the mortality index creating algorithm.



Figure 2.23. Death rate creating algorithm

Death rates creating algorithm as an exogenous index is using GDP growth rate. By multiplying it by the regression index "mortality response to changes of GDP" the changes of death rates are being calculated, taking into account GDP growth. Death rates are calculated in accordance with the death rates variance, caused by the change of GDP and constrained by long-term balanced death rates. Death rate variance is being visually characterized by the Figure 2.24.



Figure 2.24. reflects by the abstract example that in the result of the death rate variance the death rate of one age becomes the death rate of the next older age, death rates of all ages are being reduced (with GDP growth).

Balanced death rates in the long-term period constraint the death rate reduction, the same element can be used in the planning of the target population life. The index "change of death rate in accordance with GDP growth" defines speed of variance of the death rates.

The calculation of the index "change of death rate in accordance with GDP growth" is presented in formula 72. Calculation of death rate variance is presented in formula 73:

$$MKI_{t} = (IKPPT_{t} - 1) \times MR, \qquad (72)$$

where

MKI - change of death rate in accordance with GDP growth; IKPPT - growth rate of total GDP per capita; MR - mortality response to changes of GDP.

$$MKN_{VDt} = \begin{cases} 0, V = 1\\ MKI \times MK_{(V-1)D(t-1)}, MK_{(V-1)D(t-1)} \ge ILMK_{(V-1)D(t-1)}\\ (MKI \times MK_{(V-1)D(t-1)}) \lor 0, MK_{(V-1)D(t-1)} < ILMK_{(V-1)D(t-1)} \end{bmatrix}, V > 1, \\ V = \{1, 2, \dots, 99, 100 + \}, \end{cases}$$

$$(73)$$

where  $MKN_{VD}$  - death rate variance; MKI - change of death rate in accordance with GDP growth;  $MK_{VD}$  - death rates;  $ILMK_{VD}$  - balanced death rates in the long-term period; V - age.

Formula 73 reflects the death rate variance is being applied only for the death rates of the age above one year. This part of the equation is based on the assumption that the infant mortality is independent of changes of GDP. Then, for the age above one year, if death rate is higher than the balanced death rates in the long-term period (relevant group), then the variance is formed as the multiplication of the index "change of death rate in accordance with GDP growth". This helps to reduce and increase the mortality index variance along with a decrease and increase of GDP (when the death rates are above the long-term balanced indexes, i.e. less than the target or a reasonable minimum level). When the death rates are lower than the balanced death rates in the long-term period, death rates during the crisis of GDP can be only increased (as well as mortality).

Death rates meet the set changes, their calculation is presented in formula 74:

$$MK_{VD}(t) = MK_{VD}(t_{0}) + \int_{t_{0}}^{T} (-MKN_{(V-1)D} + MKN_{VD}) dt,$$
  

$$V = \begin{cases} \{1, 2, \dots, 99, 100 + \}, V \in MKN_{(V-1)D} \lor MKN_{VD} \\ \{0, 1, \dots, 99, 100 + \}, V \in MK_{VD} \end{cases},$$
(74)

where  $MK_{VD}$  - death rates;  $MKN_{VD}$  - death rate variance; V - age.

Formula 74 shows that the death rate is based on the value in the previous period, but the changes cause the rate variance. In the result of the rate variance, the relevant age the death rates will be reduced, as well as the mortality.

## 2.2.2 Education attainment sub-model

Education attainment sub-model consists of two parts: a sub-model of education attainment and sub-model of lifelong education system. The structure of the sub-model of the education attainment is presented in the Figure 2.25.





Model of education attainment is based on a unified algorithm. All levels of education involve one and the same algorithm, but the provided constants and initial levels (input data) for each level are different. Education attainment and lifelong education sub-models are based on a unified algorithm, with the difference that the education attainment sub-model involves a continuous flow from the primary to doctoral education, but in the lifelong education sub-model the number of people, continuing studies of the highest level does not depend on the number of lower-level graduates, but on the population with the lowest level of education (i.e., it the principle of the continuation of lifelong education is being ensured).

So the education attainment sub-model (Figure 2.25) reflects the Latvian education system, which begins with primary education, the secondary education follows (or vocational education), then it is possible to get the higher education or vocational education (up to bachelor's degree level), after the bachelor's degree is possible to obtain a master's degree and then - the degree of doctor.

Continuation of education is being defined by the unified algorithm, which is identical at all levels of education, but the input data is different at each level. Algorithm of continuation of education considers the labour demand and supply volumes in different education groups.

Then the education acquisition and continuation unified algorithms are considered.

### Education acquisition unified algorithms

The general scheme of the unified education acquisition algorithm is presented in Figure 2.26.



52

Education stock includes students (pupils or students, depending on the level of education), which are getting education. Flow "learning process" moves the student (pupil or student) to the next class (the course); the other incoming and outgoing flows in the stock are intuitively understandable: students (pupils or students) start their education, entering the stock and leave it, stopping the studies or getting the education. The calculation of the education acquisition stock is being described by the following equation (formula 75):

$$IA_{GJD}(t) = IA_{GJD}(t_{0}) + \int_{t_{0}}^{T} (SA_{GJD} - II_{GJD} - SP_{GJD} - AP_{(G-1)JD} + AP_{GJD}) dt,$$

$$G = \begin{cases} \{2,3,\dots,i-1,i\}, G \in AP_{(V-1)JD} \lor AP_{VJD} \\ \{1,2,\dots,i-1,i\}, G \in IA_{VJD} \lor II_{VJD} \lor SP_{VJD} \end{cases},$$
(75)

where

 $IA_{GJD}$  - mastering of education;  $SA_{GJD}$  - beginning of education mastering;  $II_{GJD}$  - acquisition of education;  $SP_{GJD}$  - interruption of education;  $AP_{GJD}$  - learning process; G - academic year.

As stated above, the flow "learning process" moves the student (pupil or student) to the next class (the course). Everybody, who have not stopped their studies or have not received education in the analysed year, move to the next class (course). So the flow is being defined by the number of the students (pupils or students), (which is analysed in the model in the stock "mastering of education"), acquisition of education or interruption of the education. The calculation of the flow "learning process" is presented in formula 76:

$$AP_{GJDt} = IA_{GJDt} - II_{GJDt} - SP_{GJDt}, G \in 1...i-1,$$

$$(76)$$

where

 $AP_{GJD}$  - learning process; IA<sub>GJD</sub> - mastering of education; II<sub>GJD</sub> - acquisition of education; SP<sub>GJD</sub> - interruption of education; G - academic year.

Formula 76 shows that the students of the last academic year (pupils or students) are not reflected in the flow "learning process", they leave educational institution in this year.

Education acquisition is dependent on the learning time, which is constant for each level of education and each field of education, and on the number of students (pupil or student), who have studied the appropriate number of years. The calculation formula of the education acquisition is shown in formula 77:

$$II_{GJDt} = \begin{cases} AI_{GJDt}, G = AI_J \\ 0, G \neq AI_J \end{cases},$$
(77)

where  $II_{GJD}$  - acquisition of education;  $IA_{GJD}$  - mastering of education;  $AI_J$  - learning time; G - academic year.

Formula 77 shows that students (pupils or students) acquire education, if they have studied the appropriate number of years. It is important to note that the model operates with a whole number of years, and the input data of the learning time should consist of whole numbers in order to obtain the adequate results.

Calculation of the interruption of the studies is slightly more complicated. The interruption of studies may have two reasons: natural population change (mortality or migration) or dropout. It is being described by formula 78:

$$SP_{GJDt} = \begin{cases} \left(AT_{GJDt} + SPmm_{GJDt}\right), IA_{GJDt} > \left(AT_{GJDt} + SPmm_{GJDt}\right) \\ IA_{GJDt}, IA_{GJDt} \le \left(AT_{GJDt} + SPmm_{GJDt}\right) \end{cases},$$
(78)

where

 $\begin{array}{l} SP_{GJD} \mbox{-} interruption of education; \\ AT_{GJD} \mbox{-} dropout; \\ SPmm_{GJD} \mbox{-} interruption of education due to mortality and migration; } \\ IA_{GJD} \mbox{-} mastering of education. \end{array}$ 

Interruption of the studies is being defined by the mortality (along with migration) and dropouts, but it cannot exceed the number of students that is reasonably understandable. This logical constraint is being represented by equation 78.

Dropouts, which are calculated from the number of students (pupils or students) and the index of dropouts, the calculation equation is presented in formula 79:

$$AT_{GJDt} = \begin{cases} \frac{IA_{GJDt} \times AK \times (1 - AKs)}{AI_J}, D = \text{Men} \\ \frac{IA_{GJDt} \times AK \times AKs}{AI_J}, D = W omen \end{cases},$$
(79)

 $AT_{GJD}$  - dropout; IA<sub>GJD</sub> - mastering of education; AKs - women dropout index; AK - level of education dropout index; AI<sub>J</sub> - learning time.

Formula 79 shows that the fixed part is being dismissed from the students, and the part differs by gender. Dropout is being evenly distributed by academic years. Dropout indexes are calculated from the statistical data, as the average indexes for levels of education and gender. Dropout indexes are not changing depending on the field of education. Women's dropout index is constant at all levels of education, and is equal to 0.377726. This means that women make around 37.8% of the total number of dismissed. Dropout indexes by educational levels are presented in Annex A4.

The index "interruption of studies due to mortality and migration" connects the learning process with demographic processes, i.e., regulates the number of students in accordance with demographic changes. This index divides the mortality and migration of the population of the corresponding age by the students, in accordance with the education structure. The calculation of the index "interruption of studies due to mortality and migration" is presented in formula 80:

$$SPmm_{DJDt} = SS_{Jt} \times SMM_{GDt}, \qquad (80)$$

 $SPmm_{GJD}$  - interruption of education due to mortality and migration;  $SS_J$  - structure of students by fields of education;  $SMM_{GD}$  - mortality and migration of students.

Student structure by fields of education is calculated on the basis of the number of students, see formula 81:

$$SS_{Jt} = \frac{\sum_{i \in G} \sum_{l \in D} IA_{GJDt}^{\ \ il}}{\sum_{i \in G} \sum_{l \in D} \sum_{k \in J} IA_{GJDt}^{\ \ ilk}},$$
(81)

where

 $SS_J$  - structure of students by fields of education;

IA<sub>GJD</sub> - mastering of education;

G - academic year;

D - gender;

J - field of education.

Mortality and migration of students is calculated from mortality and emigration of the population of the corresponding age, see formula 82:

$$SMM_{GDt} := \sum_{V=i}^{V=i+G} \left( M_{VDt}^{V} + EMI_{VDt}^{V} \right), \tag{82}$$

 $SMM_{GD}$  - mortality and migration of students.  $M_{VD}$  - mortality;  $EM_{VD}$  - emigration; V - age; G - academic year.

Formula 82 reflects that for the mortality and emigration from the appropriate age of the beginning of education (V=i) to the age of completion of education (V=i+G) the dimension is being converted into the dimension of the academic year, thus providing the synchronization of demographic and educational sub-models (in the demographic sub-model the population is being analysed by one-year age groups, but in the education sub-model - by learning (academic) years)). Mortality and emigration of the population are being calculated in the related sub-models. Formula 82 reflects the emigration index rather than migration, taking into account that if the people are leaving the country, they leave the education system, but arriving people do not automatically add the number of students.

In general, unified sub-model of the education attainment is presented in the figure 2.27.



Figure 2.27. Unified sub-model of the education attainment

From the unified sub-model elements all elements have been previously explained, except the flow "beginning of mastering of education". This flow connects the previous level of education. Visually, this connection at bachelor and master levels is being characterized, as an example, by Figure 2.28.



# Figure 2.28. Unified connection of sub-model of education attainment by the example of the levels of bachelor's and master's studies

Figure 2.28 shows that the sub-models of two levels of education attainment are connected with the education continuation sub-algorithm, which is also unified, and similarly connects all levels of education. Algorithm of continuation of education at the level of bachelor's and master's studies is presented in Figure 2.29.



Figure 2.29. Algorithm of continuation of education at the level of bachelor's and master's studies

Figure 2.29 reflects that after the completion of the level of studies the alumni are moved from the course dimension to the age dimension. Then the test of the obtained qualification follows (this is necessary as at the certain levels of education a variety of qualifications can be obtained, including insufficient to continue the education at the next level). The part of the number of graduates decide to continue further studies, this choice is being defined by the education continuation index. For the determination of the education continuation index the unique unified sub-model has been developed, which is discussed below. The model provides that the beginning of education acquisition defines the wish to continue studies, but in the base period this is a statistic value. After the decision to continue studies, according to the field of education demand structure, the field of education, in which the decision maker continues studies, is being defined. The next step is the filling of the vacant study places. If the number of graduates is less than the number of vacancies, all graduates occupy vacant places and begin the studies. Otherwise, all vacant places are occupied, but graduates are accepted in proportion to the structure of graduates.

Formula 83 demonstrates the movement of the course dimension of graduates from the level of studies to the age dimension:

$$II_{VJDt} := II_{GJDt} , \qquad (83)$$

where

 $II_{VJD}$  - education acquisition by age groups;  $II_{GJD}$  - acquisition of education.

Most often, in order to obtain the qualification, required for the further continuation of studies, it is necessary to study more than in other programs (laws and regulations determine a minimal learning time). On this basis, it can be assumed, that the number of persons, who have obtained qualification, may be selected by age, which is being described in formula 84 for bachelor's studies:

$$B_{VJDt} = II_{VJDt}, V \in n,$$
(84)

where

 $B_{VJD}$  - bachelor degree or qualification;

 $II_{VJD}$  - education acquisition by age groups.

By multiplying the number of people with the qualification by index of continuation of the studies, the number of people, who decide to continue studies at the next level, is being calculated. The example of the bachelor's degree recipients is presented in formula 85:

$$NTM_{VJDt} = B_{VJDt} \times ITK_t, \tag{85}$$

where

 $NTM_{VJD}$  - decide to continue the studies;  $B_{VJD}$  - bachelor degree or qualification; ITK - education continuation index.

Upon taking the decision the alumni at one level of education become the graduates of the next level of education. Number of graduates by gender and education is being calculated by multiplying the demand structure of the fields of education by the number of alumni at the previous level of education, thus defining the division of graduates by fields of education, formula 86:

$$ABS_{JDt} = \begin{cases} B_{GJD}, t = 0\\ NTM_{Dt} \times IP_{Jt}, t > 0 \end{cases}$$
(86)

where

 $ABS_{JD}$  - number of graduates by gender and fields of education;  $B_{GJD}$  - admission in the base period;  $NTM_D$  - decide to continue the studies;  $IP_J$  - structure of field of education demand; t - period of time.

Formula 86 equation provides that the education acquisition is not being started in the base period by the number of students (pupils or students), but the number, which is taken from the statistical data. Also, regardless of the age of admission, all graduates are admitted to the first year (class), i.e., starting the first study year.

In accordance with the number of graduates by gender and fields of education is being calculated the gender and education structure of the graduates, formula 87:

$$ABSt_{JDt} = \begin{cases} \frac{ABS_{JDt}}{\sum_{k=D} ABS_{JDt}^{i}}, \sum_{k=D} ABS_{JDt}^{i} > 0\\ \sum_{k=D} ABS_{JDt}^{i} \le 0 \end{cases}, \quad (87)$$

where

 $ABSt_{JD}$  - structure of graduates' gender and fields of education;  $ABS_{JD}$  - number of graduates by gender and fields of education.

When the gender and education structure of the graduates is being calculated, the number of graduates is being compared to a number of vacant places and the vacant places are being occupied, formula 88:

$$VSVA_{JDt} = \begin{cases} ABS_{JDt}, \sum_{i=D} ABS_{JDt}^{i} \leq VSV_{Jt} \\ ABSt_{JDt} \times VSV_{Jt}, \sum_{i=D} ABS_{JDt}^{i} > VSV_{Jt} \end{cases},$$
(88)

 $VSVA_{JD}$  - occupation of vacant study places; ABS<sub>JD</sub> - number of graduates by gender and fields of education; VSV<sub>J</sub> - vacant study places; ABSt<sub>JD</sub> - structure of gender and field of education of the graduates.

Formula 88 demonstrates that if the number of graduates is less than the number of vacant places, all graduates occupy the vacant places and begin the studies. Otherwise, all vacant places are being occupied, but graduates are admitted in proportion to the structure of graduates.

When graduates occupy the study places, they begin their studies in the first year. The dimension of the study year change is being represented in formula 89:

$$SA_{GIDt} := VSVA_{IDt}, G = 1,$$
(89)

where

 $SA_{GJD}$  - beginning of education acquisition; VSVA<sub>JD</sub> - occupation of vacant study places.

The parameters of the figure 2.29 do not explain the formation of the index of continuation of studies. For the index formation the unified sub-model is being used, which is analytically shown in Figure 2.30.



Figure 2.30. Calculation sub-model of the education continuation index

The level of the education continuation index in the base period is set in the one level with the long-term balanced index of continuation of education, but its changes determine the changes of the education continuation index. Changes are related to two groups of factors: taking into account the salary and employment rate at the analysed levels of education (i.e., actual education and next level of education).

Considering the salary level, the changes of the education continuation index are being determined in accordance with the average salaries at the levels of education.

The changes of the education continuation index, taking into account the employment rate, are related to the demand and supply of labour in relation to the analysed education. The changes of this index are calculated from the current index and the demanded index, as well as the period of the changes of index. The changes of the index of the continuation of the demanded studies reflect the correlation between the population and labour demand by educational level.

Formula 90 shows the calculation of the index of continuation of education:

$$ITK(t) = ITK(t_0) + \int_{t_0}^{T} ITKi \, dt = ILSITK + \int_{t_0}^{T} ITKi \, dt , \qquad (90)$$

where

ITK - education continuation index;

ITKi - changes education continuation index;

ILSITK - balanced education continuation index in the long-term period.

Calculation of changes of the education continuation index is presented in formula 91:

$$ITKi_{t} = \begin{cases} 1 - ITK_{t}, ITKiAL_{t} + ITKiNL_{t} + ITK_{t} > 1\\ ITKiAL_{t} + ITKiNL_{t}, 0 \le ITKiAL_{t} + ITKiNL_{t} + ITK_{t} \le 1, \\ - ITK_{t}, ITKiAL_{t} + ITKiNL_{t} + ITK_{t} < 0 \end{cases}$$
(91)

where

ITKi - changes of education continuation index;

ITK - education continuation index;

ITKiAL - changes of education continuation index in accordance with the wage level;

ITKiNL - changes of education continuation index in accordance with the level of employment.

Changes of the education continuation index cannot make the index of continuation to go beyond the borders of "0 to 1" (from "nobody is continuing studies" to "everybody continues"). It is determining various algorithms in formula 91, taking into account the incoming parameter values. In general, the objective of this algorithm is to approximate index of continuation of studies to the requirements of the market, taking into account logical constraints (from "nobody is continuing studies" to "everybody continues").

Calculation of the index of continuation of studies is presented in formula 92, taking into account the level of employment:

$$ITKiNL_{t} = \frac{ITK_{t} \times (PITKiNL_{t} - 1)}{t_{JTK}}, \qquad (92)$$

ITKiNL - changes of education continuation index in accordance with the level of employment;

ITK - education continuation index;

PITKiNL - changes of demanded education continuation index studies in accordance with the level of employment;

 $t_{\text{ITK}}$  - period of changes of education continuation index.

Formula 91 shows that in accordance with the level of employment the index of continuation of studies adjusts the index of continuation of studies in accordance with the index demand. The index demand changes immediately with the changes of labour demand and supply. But in reality, to understand that the education demand has been established, people need time. Therefore, the changes of the education continuation index, demanded in formula 91, are divided into time of changes of the education continuation index, thus ensuring the process delay in time.

The example of calculation of the demanded education continuation index, taking into account the level of employment, from the general secondary and vocational education to the first level vocational higher education is presented in formula 93:

$$PITKiNL_{t} = \frac{\frac{I_{a_{vvpvi}}}{I_{a_{plpair}}} \times \frac{N_{l_{plpair}}}{N_{l_{vvpvi}}}}{\frac{I_{a_{vvpvi}}}{I_{a_{plpair}}} \times \frac{N_{l_{plpair}}}{N_{l_{plpair_0}}}},$$
(93)

where

PITKiNL - changes of demanded education continuation index studies in accordance with the level of employment;

 $I_{a i}$  - active population with i level of education;

 $N_{l_i}$  - number of employed people with i level of education.

Formula 93 evaluates the supply and demand changes of labour with the general, vocational and first level vocational higher education. This equation is unified and is used for various levels of education. Each of the evaluations uses the supply and demand factors of the previous education. But the calculation of each level of education has the specifics of the highest levels. Usually the demand of higher levels influences the education demand at the certain level. For example, education demand at the doctoral level consists of master's, bachelor's (etc.) levels of education demand. The incoming parameters of education demand of higher levels are presented in the table 2.2.

	Active population and labour demand with the level of education:					
	general					
	vocational	secondary	vocational	Bachelor's	Master's	Doctoral
	education					
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Demand from the						
basic education to	x					
the general						
vocational education						
Demand to continue						
studies after the		Х	Х	Х	Х	Х
basic education						
Demand from the						
basic education to		х		х	х	Х
the secondary						
Demand from the						
Demand from the						
to the vocational			V			
education			Χ			
cuucation						
Demand from the						
secondary education				х	х	х
to Bachelor's degree						
Demand from						
Bachelor's degree to					Х	х
the Master's degree						
Demand from the						
Master's degree to						Х
Doctoral degree						

 Table 2.2. The parameters of education demand of higher levels

As shown in table 2.2, the reception of the secondary education is being stimulated not only by the direct demand with secondary education, but also the demand for workers with a doctoral degree etc.

The calculation of the changes education continuation index, taking into account the level of salary, is represented in formula 94:

$$ITKiAL_{t} = \frac{ITK_{t} \times (PITKiAL_{t} - 1)}{t_{JTK}},$$
(94)

where

ITKiAL - changes of education continuation index in accordance with the level of wages in accordance with the level of employment;

ITK - education continuation index;

PITKiAL - changes of demanded education continuation index in accordance with the wage level;

 $t_{\text{ITK}}$  - period of changes of education continuation index.

Formula 94 shows that the calculation of the changes of the education continuation index, taking into account the level of salary, is similar to the changes of the education continuation index,

taking into account the level of employment. The example of calculation of demanded education continuation index, taking into account the level of salary, from the general secondary and vocational education to the first level vocational higher education is presented in formula 95:

$$PITKiAL_{t} = \frac{\frac{A_{\text{plpair}}}{A_{\text{vvpvi}}}}{\frac{A_{\text{plpair}_{0}}}{A_{\text{vvpvi}_{0}}}},$$
(95)

where

PITKiAL - changes of demanded education continuation index in accordance with the wage level;

 $A_L$  - average salaries at the levels of education.

Formula 95 evaluates the salary in the groups with the general, vocational and first level vocational higher education. This equation is unified and is used for various levels of education. Formula 95 represents that the decision to continue the studies is being affected only by salaries of two levels of education: salary of the last education (current level) and the salary of the next level of education.

Figure 2.29 represents the element "structure of fields of education demand". Demand structure of fields of education is determined by the existing vacancies in the labour market at the certain level of education. The more vacancies in the field of education, the greater the education demand in this field. The calculation of the demand structure of fields of education is presented in formula 96:

$$IP_{Jt} = \begin{cases} \sum_{k=D} \sum_{n=P} VS_{DPJ_{it}}^{kn} \\ \sum_{k=D} \sum_{n=P} \sum_{l=J} VS_{DPJ_{it}}^{knl} \\ 0, \sum_{k=D} \sum_{n=P} \sum_{l=J} VS_{DPJ_{it}}^{knl} \le 0 \end{cases},$$
(96)

where

IP<sub>J</sub> - structure of fields of education demand; VS<sub>DPJ i</sub> - number of vacancies at the education level i.

Formula 96 is being applied to vocational education, vocational secondary education, vocational education after the secondary education and higher education, i.e., where there is an urgent question of the choice of field of education. This equation is not completely unified, as the model provides that the populations continue to receive highest level of education (higher education) in accordance with the already received education, i.e., doctoral and master's levels coincides with the bachelor's level of education.

#### Unified algorithm of state-financed study places

Education sub-model involves one more unified algorithm, which is not related to the education attainment by the population, but to the state policy planning in the field of education. The objective of the unified algorithm is to determine the optimal number for the labour market of the state-financed places by levels and fields of education.

The developed model provides the application of exogenously specified number of study places for the education system, as well as modelling of the number of the study places in

accordance with the market situation. The type of determination of study places used in the model depends on the choice and activities of the model operator in the developed interface. The essence of the algorithm of determination of study places is being described in the Figure 2.31.



Figure 2.31. The essence of the algorithm of determination of study places

As the Figure 2.31 shows, there are two types of study places: provided vacant study places and optimized vacant study places. Depending on the interface parameter "type of determination of study places", status element "vacant study places" accepts one of these values. The data in the determination algorithm of the study places is not modified or processed, this algorithm represents the logical choice of the operator and it has a technical support function of the model, so that the technical code of the algorithm is not described in the form of mathematical equation.

Figure 2.31 shows the parameter "vacant study places" (which has already been occurring, for example in formula 88). This is a key indicator, which determines the number of vacant study places in the model. If it is equal to "provided number of vacant study places", the model reflects the effect of the exogenously provided number of study places on the education system and the labour market. In this case, the model does not apply any algorithms for determination of the study places.

If the parameter "vacant study places" accepts the values of the parameter "optimized vacant study places", the model is based on the optimization algorithm of the study places, which is presented in Figure 2.32.



Figure 2.32. Optimization algorithm of the study places

Optimization algorithm of the study places is complex and complicated. On the one hand, there is the effect of the labour market demand and supply on the study places, but on the other hand - the opportunities and the desire of the population to participate in educational processes. Let us gradually review it.

The ration of labour demand and supply (ratio of labour demand and active population by fields of education) is being calculated on the basis of the labour demand distributed by fields and levels of education (defined in subchapter 2.1) and the active population (defined in subchapter 2.2.3) (this equation is unified, and will be applied to all levels of education). The average ratio in the fields of education, which have the ratio of labour demand and population lower than average (critical fields of education) are being determined on the basis of the ratio by fields of education and average ratio. If these fields do not have state-funded study places, the new study places (the number of study places is being defined by the index "initial number of study places in the new fields") are being opened. If there are state-financed study places (increased for critical fields, decreased for other fields). By summing up the advanced study places and study places in the new fields, the summed up advances study places. Its objective is to help students to go in the fields with the greatest shortage of labour.

The second part of the algorithm compares summed up advanced study places and the number of applicants (applications). Summed up advanced study places are optimized in accordance with the number of applicants, that is, if the number of summed up advanced study places exceeds the number of applications, the inefficient places are being liquidated. Thus, the number of study places comes after the number of applications (which is also a function of the population). However, with an increase in applications, the number of study places does not grow, as the maximum number of study places is being determined by the labour supply and demand, but the minimum number - by the population.

To ensure the functionality of the model "number of optimized study places" is compared with the existing optimized vacant study places, the flow "changes of study places" is being created, which determines the stock "optimized vacant study places".

Upon explanation of the essence of algorithm of optimization of study places let us look at the forming equations. The first forming equation is the ratio of labour demand and active population by fields of education, formula 97:

$$LSIA_{Jt} = \begin{cases} \sum_{k=L} \sum_{n=P} LS_{LJPt}^{kn} \\ \sum_{k=Vg} \sum_{n=D} \sum_{l=P} I_{VgDPJEt}^{knl}, \sum_{k=Vg} \sum_{n=D} \sum_{l=P} I_{VgDPJEt}^{knl} > 0 \\ -1, \sum_{k=Vg} \sum_{n=D} \sum_{l=P} I_{VgDPJEt}^{knl} \le 0 \end{cases}, E = active, \qquad (97)$$

where

 $LSIA_J$  - ratio of labour demand and active population by fields of education;  $LS_{JLP}$  - labour demand by fields, levels of education and occupations;  $I_{VgDPJE}$  - population.

Formula 97 uses the existing parameters in the model, sums them up to the necessary dimension - by fields of education, selecting from the population only the economically active population (i.e., only labour power). If the labour power of the certain field of education does not have any representative in order to ensure the functionality of the model, the value of -1 is being assigned to the ratio. Later it will also allow dividing situations when there is no labour demand from those cases when there is no labour.

Upon the calculation of the ratio of labour demand and population, the average ratio of labour demand and population is being determined, formula 98:

$$\overline{LSIA_t} = \frac{\sum_{k \in J} (LSIA_{Jt} \lor 0)}{\sum_{k \in J} x_k},$$
(98)

where

 $x_k = \begin{cases} 1, & (LSIA_{k \in J} \lor 0) > 0 \\ 0, & (LSIA_{k \in J} \lor 0) = 0 \end{cases}$ 

 $\label{eq:LSIA-average} LSIA \mbox{-} average ratio of labour demand and active population; \\ LSIA_J \mbox{-} ratio of labour demand and active population by fields of education.$ 

As formula 98 shows, the average calculation involves only the fields of education with labour, i.e. ratio of labour demand and active population by fields of education is greater than zero.

The critical fields of education are being determined on the basis of the ratio of labour demand of active population, formula 99:

$$KIJ_{J_{t}} = \begin{cases} 1, LSIA_{J_{t}} = -1 \\ 0, LSIA_{J_{t}} = 0 \\ 1, \frac{LSIA_{J_{t}}}{LSIA_{t}} \ge 1 \\ 0, \frac{LSIA_{J_{t}}}{LSIA_{t}} < 1 \end{cases}$$
(99)

where

KIJ<sub>J</sub> - critical fields of education;

LSIA - average ratio of labour demand and active population;

LSIA<sub>J</sub> - ratio of labour demand and active population by fields of education.

Formula 99 shows that the parameter assigns the value 1 to the critical fields of education; the other fields are being assigned the value of 0. Formula 99 indicates that the critical fields of education are those without labour, as well as the ratio of labour demand and active population by fields of education is greater than or equal to the average ratio. If there is no labour demand or ratio of active population by fields of education is less than the average ratio, the fields are not critical.

By comparing the critical fields of education with the study places by fields, the decision on the opening of study places in the new fields (formula 100) or advancement of current number of places (formula 101) is being taken:

$$JSVA_{Jt} = \begin{cases} (OVSV_{Jt} = 0) \land (KIJ_{Jt} > 0) \Rightarrow JJVS \\ 0 \end{cases},$$
(100)

where JSVA<sub>J</sub> - opening of study places in the new fields; OVSV<sub>J</sub> - optimized vacant study places; KIJ<sub>J</sub> - critical fields of education; JJVS - initial number of study places in the new fields.

Formula 100 shows that, if the field of education does not have optimized vacant study places, but at the same time, this is critical, the new study places are being opened in the field (the number is being defined by the index "the initial number of the study places in the new fields"). In all other cases, the opening of new study places in the field is not taking place:

$$USV_{Jt} = \begin{cases} \left[ OVSV_{Jt} \times SVIR \right], KIJ_{Jt} = 1 \\ \left[ \frac{OVSV_{Jt}}{SVIR} \right], KIJ_{Jt} = 0 \end{cases},$$
(101)

where USV<sub>J</sub> - advanced study places; OVSV<sub>J</sub> - optimized vacant study places; SVIR - constraint of the changes of study places; KIJ<sub>J</sub> - critical fields of education.

Formula 101 shows that if the number of study places in the critical fields of education is being increased according to the index "constraint of the changes of study places". The number of study places in all other fields is being reduced.

Upon the opening of study places in the new fields and advancement of the existing study places, it is necessary to determine its total effect on the study places (see formula 102):

$$SUSV_{Jt} = JSVA_{Jt} + USV_{Jt}, (102)$$

where

 $SUSV_J$  - summed up advanced study places; JSVA<sub>J</sub> - opening of study places in the new fields; USV<sub>J</sub> - advanced study places.

Summed up advanced study places compose the overall result of the opening of new study places in the fields and advancement of the existing study places.

Summed up advanced study places reflect the labour market demands for the study places. The situation, when the population may be incompatible with the market demands, is possible; in this case, the market may demand more population than can be trained. In this case, it is necessary to reduce the inefficient places. To achieve this, the summarized advanced study places are being compared and optimized according to the number of applications at the level of education.

Formula 103 reflects the calculation of the number of applications at the level of education:

$$PS_t = \sum_{k \in D} NTM_{Dt}^{k} , \qquad (103)$$

where PS - number of applications; NTM<sub>D</sub> - decide to continue studies.

The number of applications at the level of education is the total number of people who decide to continue studies at the next level of education, their total sum. Formula 104compares it with the summed up advances study places:

$$PSSVA_{t} = \left(\frac{PS_{t}}{\sum_{k \in J} SUSV_{Jt}^{k}}\right) \wedge 1, \qquad (104)$$

where

PSSVA - total ratio of number of applications and study places;

PS - number of applications;

 $SUSV_{\mbox{\scriptsize J}}$  - summed up advanced study places.

Formula 104 calculates the ratio of the total number of applications and the summed up advanced study places. In accordance with the equation, this ratio is constrained and cannot exceed 1. This constraint is being used in formula 105 in order to reduce the number of study places if the number of applications is less than the number of study places:

$$OSV_{Jt} = [SUSV_{Jt} \times PSSVA_t], \tag{105}$$

where

OSV<sub>J</sub> - number of optimized study places; SUSV<sub>J</sub> - summed up advanced study places; PSSVA - total ratio of number of applications and study places.

Formula 105 shows that the optimized number of study places is being calculated on the basis of the summed up advanced study places, by adjusting them and taking into account number of applications for studies.

Optimized number of study places by levels of education is integrated into a single matrix and compared with optimized vacant study places, by determining the changes of study places, formula 106:

$$OSVI_{JLt} = \left[ OSV_{J_{1}t} \right]_{l} \cup \left[ OSV_{J_{2}t} \right]_{2} \cup \dots \cup \left[ OSV_{J_{i}-1t} \right]_{i-1} \cup \left[ OSV_{J_{i}t} \right]_{i} - OVSV_{JLt}, \quad (106)$$

where

 $OSVI_{JL}$  - changes of study places;  $OSV_{J_i}$  - number of optimized study places at the education level *i*;  $OVSV_{JL}$  - optimized vacant study places.

The changes of study places is a flow that determines the changes of the stock of optimized vacant study places, see formula 107:

$$OVSV_{JL}(t) = OVSV_{JL}(t_0) + \int_{t_0}^{T} (OSVI_{JL}) dt$$
, (107)

OVSV<sub>JL</sub> - optimized vacant study places; OSVI<sub>JL</sub> - changes of study places.

Optimized vacant study places are used for the needs of the model (for example, by selecting the level of education in formula 111), and it is an important element while planning the state policy in the field of education.

The above presented unified sub-models are being applied to all levels of education. Individual educational levels are using the specific elements of the sub-models, which are considered below.

# The specifics of the algorithm of the basic education, vocational education with basic education

The first specific element of the sub-model is related to basic education and vocational education with basic education. This is the initial level of education. The model provides that there is no education prior to initial level. Accordingly, the sub-model cannot realize the unified algorithm of continuation of education. The model provides that the general basic education is being started by all people who reach the age of 7. This is the first specific sub-model's element.

The sub-algorithm of the basic education, vocational education with the basic education is being implemented not only in the formation of the basic education, but also in the formation of vocational education with the basic education. People, who fail to complete the basic education, start vocational education, so the sub-model should provide a possibility of changing of basic education to vocational education with basic education. It is the second specific sub-model's element. It is being described by Figure 2.33.



Figure 2.33. Features of the sub-model of the basic education, vocational education with basic education

As the Figure 2.33 shows, the sub-model of the basic education, vocational education with basic education has the specific start of the education acquisition, equal to 7-year-old population, and the additional flow "change of basic education to the vocational education with basic education" with the determinant factors.

The additional flow changes the calculation equation of the stock, the new look is presented in formula 108:

$$IA_{GJD1}(t) = IA_{GJD}(t_{0}) + \int_{t_{0}}^{T} (SA_{GJD} - AvpNap_{GJD} - II_{GJD} - SP_{GJD} - AP_{(G-1)JD} + AP_{GJD}) dt$$

$$G = \begin{cases} \{2,3,...,i-1,i\}, G \in AP_{(V-1)JD} \lor AP_{VJD} \\ \{1,2,...,i-1,i\}, G \in IA_{VJD} \lor II_{VJD} \lor SP_{VJD} \lor AvpNap_{GJD} \end{cases},$$
(108)

where

 $IA_{GJD1}$  - mastering of basic education or vocational education with basic education; AvpNap<sub>GJD</sub> - change of basic education to the vocational education with basic education;  $SA_{GJD}$  - beginning of mastering of education;  $II_{GJD}$  - acquisition of education;  $SP_{GJD}$  - interruption of education;  $AP_{GJD}$  - learning process; G - academic year.

The calculation equation of the additional flow "change of basic education to the vocational education with basic education" is presented in formula 109:

$$AvpNap_{GIDt} = Pvp_{GIDt} - Iap_{GIDt}, \qquad (109)$$

where

 $AvpNap_{GJD}$  - change of basic education to the vocational education with basic education;  $Pvp_{GJD}$  - abandon basic education;

 $\ensuremath{\text{Iap}_{\text{GJD}}}\xspace$  - mastering of vocational education with basic education.

The key role of the additional flow is to change the general basic education to vocational education with basic education. The number of students, who leave studies, is being deprived from the total number of students, and the same number of people is being added to the number of students receiving vocational education, which is generally specified in formulas 108 and 109.

#### Specifics of the algorithm of continuation of studies after the basic education

Upon acquisition of the basic education there is a choice - to continue the general secondary education or vocational education (vocational secondary education). This choice is not provided by the unified model. To realize this principle in the sub-model, the following algorithm has been developed: the demand for continuation of basic education is being calculated, and then the demand for continuation of secondary education is being calculated. The demand for continuation of vocational education (vocational secondary education) is being calculated as the difference between the demand for continuation of education after basic education and secondary education. Visually, this algorithm is being reflected in Figure 2.34.



Figure 2.34. Continuation of education after basic education

Figure 2.34 by itself repeats the unified algorithm that is reflected differential in Figure 2.29. Continuation of education after basic education is being distinguished by the fact that after the completion of basic education it can be continued in two ways: general and vocational education. Algorithm of vocational education fully repeats the unified algorithm, so it is not widely viewed. Algorithm of continuation of education after basic education differs from the unified algorithm only by the following elements: decision on continuation of studies in order to receive general secondary education, decision on continuation of studies in order to receive general secondary education) and filling of vacant study places. Let us look at the calculation: first reviewed parameter is the decision on continuation of studies in order to receive general secondary education, formula 110:

$$NTMv_{VDt} = NTMpp_{Dt} \times ITKv_t, \qquad (110)$$

where

 $NTMv_{VD}$  - decision on continuation education in the general secondary education;  $NTMpp_D$  - decision on continuation education upon basic education attainment; ITKv - education continuation index in general secondary education.

By comparing formula 110 with unified formula 85, it is evident that it maintains the principle, which is being applied to the index of continuation of education, but in this case, the index of continuation of education separates the acquisition of general secondary education from the vocational education (vocational secondary education). The index of continuation of education is being calculated by using the unified equations.

The calculation of the index "decision on continuation education in the vocational education (vocational secondary education)" is presented in formula 111:

$$NTMa_{JDt} = (NTMpp_{Dt} - NTMv_{Dt}) \times IPa_{Jt}, J \in \{141, 142, 211, \dots, 862, 863\},$$
(111)

NTMa<sub>JD</sub> - decision on continuation education in the vocational education (vocational secondary education);

NTMpp<sub>D</sub> - decision on continuation education upon basic education attainment;

 $NTMv_D$  - decision on continuation education in general secondary education;

 $\ensuremath{IPa_J}\xspace$  - demand of fields of education for vocational education and vocational secondary education.

Formula 111 shows that vocational education (vocational secondary education) fields end with the penultimate education code. Later, by summing up the vocational education (vocational secondary education) and general secondary education, one flow combines all fields of education (formula 63).

According to algorithms presented in Figure 2.34, the parameter "Occupation of vacant study places" is being changed, its calculation is presented in formula 112:

$$VSVA_{JDt} = \begin{cases} ABS_{JDt} \bigcup \text{NTM} \text{v}_{\text{Dt}}, \sum_{i=D} ABS_{JDt}^{i} \leq VSV_{Jt} \\ ABSt_{JDt} \times VSV_{Jt} \bigcup \text{NTM} \text{v}_{\text{Dt}}, \sum_{i=D} ABS_{JDt}^{i} > VSV_{Jt} \end{cases},$$
(112)

where

VSVA<sub>JD</sub> - occupation of vacant study places;

ABS<sub>JD</sub> - number of alumni by gender and fields of education;

NTMv<sub>D</sub> - decision on continuation education in general secondary education;

VSV<sub>J</sub> - vacant study places;

ABSt<sub>JD</sub> - structure of alumni' gender and fields of education.

As shown in formula 111, by comparing with the unified formula 88, instead of one direction in education the two-direction combination of education is being used. Additionally, formula 111 shows that, regardless of the number of study places, everybody who decides to study is admitted for general secondary education.

The optimization algorithm of the general secondary vocational education and secondary vocational education also has its own specifics. The number of study places in vocational education (vocational secondary education) is being calculated according to unified algorithm, but its number in general secondary education is equal to the number of graduates (everybody who decides to study is admitted for general secondary education). Thus, the algorithm of general secondary vocational education contains only one equation, which differs from the unified algorithm. And this equation combines the optimized number of study places in vocational education (vocational secondary education) and general secondary education, formula 113:

$$OSVva_{Jt} = OSVv_t \cup OSVa_{Jt} = \left(\sum_{i=D} \left(NTM v_{Dt}^{i}\right)\right) \cup OSVa_{Jt},$$
(113)

where

OSVva<sub>J</sub> - optimized number of study places in the secondary, vocational and secondary vocational education;

OSVv - optimized number of study places in the general secondary education;

OSVa<sub>J</sub> - optimized number of study places in the vocational and secondary vocational education;

NTMv<sub>D</sub> - decision on continuation education in general secondary education.
As formula 113 shows, the optimized number of study places in general secondary vocational education and secondary vocational education is a combination of study places in general secondary and vocational education (vocational secondary education). The number of study places in vocational education is being calculated by the means of unified algorithm, while in general secondary education it is equal to the number of graduates (everybody who decides to study is admitted for general secondary education) by summing up the number of graduates (by summing up, eliminating the gender dimension).

#### Specifics of the algorithm of acquisition of the higher education

The unified algorithms are being used for acquisition of higher education (academic education and second-level vocational education), as well as other levels of education. Acquisition of higher education has its own specifics. First of all, higher education is divided into two parts: Bachelor's and Master's studies. Both parts use unified algorithms, wherefore only the corresponding specific calculations are being presented here, including the division of study places between bachelor's and master's levels and private funding of the studies.

The algorithm of division of study places between bachelor's and master's levels is based on the principle that the bachelor's and master's study places are related in the fixed proportion. Accordingly, by multiplying the total number of study places by constant index, it is possible to determine the number of bachelor's and master's study places. This index in the model is named as "Bachelor's index", and reflects the ratio of the number of the places of bachelor's level in the total number of study places in higher education.

Bachelor's index is used in the unified algorithm, where the calculations are related to study places: opening of study places in new fields, advanced study places and filling of vacant study places (they are defined in the unified equations, formulas 100, 101 and 88). Calculation of opening of study places in new fields and advanced study places for the bachelor's level is presented in formulas 114 and 115:

$$JSVA_{Jt} = \begin{cases} (OVSV_{Jt} = 0) \land (KIJ_{Jt} > 0) \Rightarrow JJVS \times BK \\ 0 \end{cases},$$
(114)

where

JSVA<sub>J</sub> - opening of study places of the new field; OVSV<sub>J</sub> - optimized vacant study places; KIJ<sub>J</sub> - critical fields of education; JJVS - initial number of study places of the new field; BK - bachelor's index.

$$USV_{Jt} = \begin{cases} [OVSV_{Jt} \times SVIR \times (1 - BK)], KIJ_{Jt} = 1\\ \left[\frac{OVSV_{Jt} \times (1 - BK)}{SVIR}\right], KIJ_{Jt} = 0 \end{cases},$$
(115)

where

USV<sub>J</sub> - advanced study places; OVSV<sub>J</sub> - optimized vacant study places; SVIR - limit of changes of study places; KIJ<sub>J</sub> - critical fields of education; BK - bachelor's index. By comparing the equations (formulas 100 and 101) in the unified algorithm with the equations of the bachelor's level (formulas 114 and 115), it is evident that they differ only by the fact that the bachelor's level values are being multiplied by the bachelor's index.

The calculation of opening of study places in new fields and advanced study places for the master's level is presented in formulas 116 and 117:

$$JSVA_{Jt} = \begin{cases} (OVSV_{Jt} = 0) \land (KIJ_{Jt} > 0) \Longrightarrow JJVS \times (1 - BK) \\ 0 \end{cases},$$
(116)

where

JSVA<sub>J</sub> - opening of study places of the new field; OVSV<sub>J</sub> - optimized vacant study places; KIJ<sub>J</sub> - critical fields of education; JJVS - initial number of study places of the new field; BK - bachelor's index.

$$USV_{Jt} = \begin{cases} [OVSV_{Jt} \times SVIR \times (1 - BK)], KIJ_{Jt} = 1\\ \left[\frac{OVSV_{Jt} \times (1 - BK)}{SVIR}\right], KIJ_{Jt} = 0 \end{cases},$$
(117)

where

USV<sub>J</sub> - advances study places; OVSV<sub>J</sub> - optimized vacant study places; SVIR - limit of changes of study places; KIJ<sub>J</sub> - critical fields of education; BK - bachelor's index.

The same formula is being used for the master's level, but instead of the bachelor's index the ratio of places in the master's studies is being used, which is being calculated by subtracting from 1 the bachelor's index.

Filling of vacant study places at the bachelor's and master's levels is accordingly presented in formulas 118 and 119:

$$VSVA_{JDt} = \begin{cases} ABS_{JDt}, \sum_{i=D} ABS_{JDt}^{i} \leq (VSV_{Jt} \times BK + SVP_{J}) \\ ABSt_{JDt} \times (VSV_{Jt} \times BK + SVP_{J}), \sum_{i=D} ABS_{JDt}^{i} > (VSV_{Jt} \times BK + SVP_{J}), \end{cases}$$
(118)

where

VSVA<sub>JD</sub> - occupation of vacant study places;

ABS<sub>JD</sub> - number of graduates by gender and fields of education;

VSV<sub>J</sub> - vacant study places;

ABSt<sub>JD</sub> - structure of alumni' gender and fields of education;

BK - bachelor's index;

SVP<sub>J</sub> - private financing of the study places.

$$VSVA_{JDt} = \begin{cases} ABS_{JDt}, \sum_{i=D} ABS_{JDt}^{i} \leq (VSV_{Jt} \times (1 - BK) + SVP_{J}) \\ ABSt_{JDt} \times (VSV_{Jt} \times (1 - BK) + SVP_{J}), \sum_{i=D} ABS_{JDt}^{i} > (VSV_{Jt} \times (1 - BK) + SVP_{J}), \end{cases}$$
(119)

where  $VSVA_{JD}$  - occupation of vacant study places;  $ABS_{JD}$  - number of graduates by gender and fields of education;  $VSV_J$  - vacant study places;  $ABSt_{JD}$  - structure of alumni' gender and fields of education; BK - bachelor's index;  $SVP_J$  - private financing of the study places.

By comparing the equations (formula 88) in the unified algorithm with the bachelor's and master's level equations, it is evident that they differ by the use of the index use of private financing of the studies. The specifics of the use of bachelor's index is being described above, but private financing of the studies is being described below.

The private financing of the studies reflects the difference between the number of admitted students and state-financed study places, its calculation equations at the bachelor's and master's levels are presented in formulas 120 and 121:

$$SVP_J = \sum_{i=D} B_{JDt0}^{i} - VSV_{Jt0} \times BK , \qquad (120)$$

where

SVP<sub>J</sub> - private financing of the study places; B<sub>GJD</sub> - admission in the base period; VSV<sub>J</sub> - vacant study places; BK - bachelor's index.

$$SVP_{J} = \sum_{i=D} B_{JDi0}^{i} - VSV_{Jt0} \times (1 - BK),$$
 (121)

where SVP<sub>J</sub> - private financing of the study places; B<sub>GJD</sub> -admission in the base period; VSV<sub>J</sub> - vacant study places; BK - bachelor's index.

The model provides that the private financing of the study places in the base year remains constant throughout the forecast period. Private financing of the study places in Latvia at this moment are weakly connected with market requirements for the occupations and fields of education, but is based on the population preference for the "prestigious" occupations. Economically unjustified determination of occupations and fields of education in the private financing does not allow modelling this process.

## Specifics of the sub-model of the acquisition of lifelong education

Sub-model of acquisition of lifelong education uses the sub-model unified algorithms. These two sub-models have the differences: lifelong education model is modelling the acquisition of education from the general secondary, vocational and vocational secondary education up to bachelor's level; indexes of continuation of education in the lifelong education are more stable than in the acquisition of the basic education: the basis of graduates in the lifelong education is not being formed by the graduates of the previous level of education, but by the population with previous level of education.

Limitation of the lifelong education sub-model functionality is related to the specifics of the levels of education: for example, the doctoral education is not a mass education, so it does not make sense to include this level of education in the lifelong education. The higher education in the labour market is not statistically distributed in the bachelor's and master's fields of education, so the continuation of education at Master's level is not included in the lifelong education, as the graduates and applicants at this level are analysed in one group both in the model and in the labour market. It is also useless to build a system of continuation of education for persons who have not attained the basic education. These persons usually continue working by simple occupations that do not require specific knowledge.

Indexes of continuation of education in the lifelong education are more stable than in the acquisition of the basic education. The pupils and students with the prior education are actively responding to market requirements, so the indexes of continuation of education due to the demand can vary significantly. The situation is the opposite in the lifelong education system: the elderly person should have the great motivation to begin mastering a new occupation. By conducting the lifelong education system research, it is being determined that a certain part of the population is still participating in lifelong education system. By dividing the population, participating in lifelong education system. This index supplements the calculation algorithm of the education of continuation index in the lifelong education.

The equation of changes of the index of continuation of education is being modified in the lifelong education algorithm, formula 122:

$$ITKi_{t} = \begin{cases} \frac{1 - ITK_{t}, \frac{ITKiAL_{t} + ITKiNL_{t}}{2} + ITKiIT_{t} + ITKiIT_{t} + ITK_{t} > 1}{2} \\ \frac{ITKiAL_{t} + ITKiNL_{t}}{2} + ITKiIT_{t}, 0 \leq \frac{ITKiAL_{t} + ITKiNL_{t}}{2} + ITKiIT_{t} + ITK_{t} \leq 1, \quad (122) \\ - ITK_{t}, \frac{ITKiAL_{t} + ITKiNL_{t}}{2} + ITKiIT_{t} + ITK_{t} < 0 \end{cases}$$

where

ITKi - changes of education continuation index;

ITK - education continuation index;

ITKiAL - changes of education continuation index, in accordance with the wage level;

ITKiNL - changes of education continuation index, in accordance with the level of employment;

ITKiIT - changes education continuation index, in accordance with long-term trends.

By comparing the equation of changes of education continuation index in the lifelong education (formula 122) with the unified equation (formula 91), it is evident that lifelong education equation is supplemented by the changes of the index of continuation of education, taking into account the long-term trends. In addition, the ratio of the changes of the index, taking into account the salaries and employment rates, has been reduced twice, it has an average level of impact.

On the basis of the unified equations, reflected in formula 122, all elements, except changes of the index of continuation of education have been calculated, taking into account the long-term trends, which calculation equation has not previously been described, but is presented in formula 123:

$$ITKiIT_{t} = \frac{ILSITK - ITK_{t}}{t_{JTK}},$$
(123)

ITKIIT - changes of index of continuation of studies, in accordance with long-term trends; ILSITK -education continuation index in the long-term period;

ITK - index of continuation of studies;

 $t_{\text{ITK}}$  - period of changes of education continuation index.

As formula 123 shows, the changes of education continuation index in accordance with the long-term trends are moving closer the education continuation index in the long-term period to the balanced education continuation index.

The basis of the graduates in the lifelong education is not being formed by the alumni of the previous level, but the population with previous level of education, which is presented in formula 124:

$$NTM_{VeDPJEt} = I_{VeDPJEt} \times ITK_t, \qquad (124)$$

where

 $NTM_{VgDPJE}$  - decide to continue the studies;  $I_{VgDPJE}$  - population; ITK - education continuation index.

In comparison with formula 85, formula 124 shows that the decision to continue education is not taken by the alumni of the previous level of education graduates, but population, analysed in the corresponding educational group. Population analysis group, its formation and calculation are investigates in the next subsection (2.2.3 subsection, labour analysis sub-model).

Formula 124 shows that the population is being analysed in the wider dimensions, in order to move to the unified equations, it is necessary to reduce the number of dimensions, which is performed in formula 125:

$$ABS_{JDt} = \sum_{k \in Vg} \sum_{i \in P} \sum_{l \in J} \sum_{n \in E} \left( NTM_{VgDPJEt}^{ki \ln} \right) \times IP_{Jt} , \qquad (125)$$

where

 $ABS_{JD}$  - number of applicants by gender and fields of education; NTM<sub>D</sub> - decide to continue the studies; IP<sub>J</sub> - structure of the field of education demand; t - time period.

Formula 125 shows that the wide groups of population are summed up to the necessary gender dimension. Then the equation repeats the unified formula 86 by its form and substance, i.e. graduates have the determined field of education.

Due to the fact that the lifelong education sub-model is associated, it is being added to the basic education sub-model, they together provide the filling of study places. Filling of study places in basic education is presented in formula 88. Lifelong education system is filling the unfilled study places in the basic education. Equation of filling of vacant study places in lifelong education is presented in formula 126:

$$VSVAM_{JDt} = \begin{cases} ABSM_{JDt}, \sum_{i=D} ABSM_{JDt}^{i} \leq \left(VSV_{Jt} - \sum_{i=D} VSVA_{JDt}^{i}\right) \\ ABStM_{JDt} \times \left(VSV_{Jt} - \sum_{i=D} VSVA_{JDt}^{i}\right), \sum_{i=D} ABSM_{JDt}^{i} > \left(VSV_{Jt} - \sum_{i=D} VSVA_{JDt}^{i}\right), (126) \end{cases}$$

VSVAM<sub>JD</sub> - occupation of vacant study places in the lifelong education; ABSM<sub>JD</sub> - number of applicants by gender and fields of education in the lifelong education; VSV<sub>J</sub> - vacant study places; VSVA<sub>JD</sub> - occupation of vacant study places;

ABStM<sub>JD</sub> - structure of alumni' gender and fields of education in the lifelong education.

By comparing the equation of filling of vacant study places in lifelong education (formula 126) with the unified equation (formula 88), it is evident that the system of lifelong education provides the study places, which have not been filled in the basic education.

In comparison with the basic education system, the specifics of the lifelong education system is that the lifelong education system does not contain dominant age group. Accordingly, it is not possible to determine the average age of alumni on the basis of the duration of the studies (as is done in the basic education system). The age of graduates in the lifelong education system is being determined in accordance with the age of alumni. The acquisition of education by age groups in the lifelong education systems is presented in formula 127:

$$II_{V_gJDt} = \sum_{k \in G} II_{GJDt}^{\ \ k} \times AbsVSt_{V_gt}, \qquad (127)$$

where

 $II_{VgJD}$  - acquisition of education by age groups;  $II_{GJD}$  - acquisition of education; AbsVSt<sub>Vg</sub> - age structure of alumni.

Acquisition of education by age groups in the lifelong education is the multiplication of the number of educated people by the age structure of alumni. The number of educated people is being calculated on the basis of the unified equation (formula 79), but the calculation of age structure is presented in formula 128:

$$AbsVSt_{Vg+1t} := AbiVSt_{Vgt}, Vg \in \{vg15\_19..vg55\_59\},$$
 (128)

where

 $AbsVSt_{Vg}$  - age structure of alumni;  $AbiVSt_{Vg}$  - age structure of graduates.

As shown in formula 128, upon the acquisition of education the alumni change the age groups, i.e., move to the next group, the older group of five-years old. This equation reflects that the studies require time and the graduates enter the educational institution at one age (age group), but graduate the older age (next age group).

The age structure of graduates is being calculated on the basis of graduates, formula 129:

$$AbiVSt_{Vgt} = \frac{\sum_{k \in D} \sum_{i \in P} \sum_{l \in J} \sum_{n \in E} NTM_{VgDPJEt}^{kinl}}{\sum_{k \in D} \sum_{i \in P} \sum_{l \in J} \sum_{n \in E} \sum_{m \in Vg} NTM_{VgDPJEt}^{kinlm}},$$
(129)

where  $AbiVSt_{Vg}$  - age structure of graduates;  $NTM_D$  - decide to continue the studies.

As formula 129 shows, the age structure of graduates is being calculated on the basis of the population who have decided to continue education in the lifelong education system, by summing up the population by age groups and dividing it by the total population.

The calculation equation of age structure of the graduates is the final specific equation in the education system. All other model calculations in the education system are realised in accordance with the unified equations (formulas 77 - 97).

# 2.2.3. Working-age population stock-taking sub-model



Working-age population stock-taking sub-model structure is presented in Figure 2.35.

Figure 2.35. Working-age population stock-taking sub-model structure

Figure 2.35 reflects that the working-age population stock-taking sub-model structure consists of various algorithms. The essence of the sub-model is the following: the working-age population is divided into five groups according to the level of attained education: each of which includes a unified population and structural analysis sub-model. All groups of population reduce beyond working age, or due to natural mortality. Natural mortality amounts are determined by demographic sub-model, but their division by the population analysis groups is determined by the algorithm of the mortality division of the working-age population by levels of education.

Changes of the population age structure are determined by the algorithm of the aging division of the working-age population, which synchronizes the demographic sub-model and working-age population stock-taking sub-model. The integral part of the aging algorithm is the retirement algorithm, which characterizes the population aging up to the age when they leave the labour market, i.e. retire. Retirement algorithm is the specific part of the aging algorithm, so it is not split into a separate algorithm, but is investigated by other algorithms.

The acquisition of education causes the changes of population in the analysis groups. Education attainment increases the population in the group according to the obtained education and reduces the population in the group in accordance with the previous education. This is determined by education technical algorithms. They have a technical function - to shift population from one group to another, as the education attainment is analysed in the education attainment sub-model.

Upon acquisition of education the population can change or choose the occupation. Choice of occupation upon the acquisition of education algorithm determines the change or choice of occupation.

Working-age population stock-taking sub-model also reflects the operation of international migration and labour force participation algorithms. International migration algorithms can both increase and reduce the population. Labour force participation algorithms change the economic activity status of the population, i.e., change the population structure.

These algorithms have a unified nature, they are applicable to all population groups. Further the operation of the unified algorithm is investigated.

#### **Population analysis algorithm**

The overall scheme of the population analysis algorithm is presented in Figure 2.36.



# Figure 2.36. Population analysis sub-model

As the Figure 2.36 shows, by acquiring education and occupation, the population is admitted for analysis in the unified algorithm in accordance with the level of education. Population age structure is changed when the flow "aging" indicates it. Beyond the retirement age, the population is taken away from the analysis. The changes of population and structural changes are possible, taking into account the migration processes, mortality, and acquisition of higher levels of education. The change of occupation is analysed in the separate flow, which is related to the change of occupation model in the market equilibrium module

The calculation of the stock of the population analysis is presented in formula 130:

$$I_{\text{VgDPJE}}(t) = I_{\text{VgDPJE}}(t_0) + \int_{t_0}^{T} \left( UP_{\text{VgDPJE}} - UN_{\text{VgDPJE}} - ISSI_{\text{VgDPJE}} - PMTB_{\text{VgDPJE}} - NOV_{(Vg-1)\text{DPJE}} + NOV_{Vg\text{DPJE}} \right) dt$$

$$Vg = \begin{cases} \{20_{24}, \dots, 65_{69}, 70_{74}\}, Vg \in NOV_{(Vg-1)DPJE} \lor NOV_{VgDPJE} \\ \{15_{19}, 20_{24}, \dots, 70_{74}\}, Vg \in I_{VgDPJE} \lor UP_{VgDPJE} \lor UN_{VgDPJE} \lor ISSI_{VgDPJE} \lor PMTB_{VgDPJE} \end{cases}, (130)$$

where  $I_{VgDPJE}$  - population;  $UP_{VgDPJE}$  - admission for analysis;  $UN_{VgDPJE}$  - removal from analysis;  $ISSI_{VgDPJE}$  - change of population and structure;  $PMTB_{VgDPJE}$  - change of occupation in the market module;  $NOV_{VgDPJE}$  - aging; Vg - age group.

Formula 130 reflects the essence of the Figure 2.36. It is important to note that the 130 formula indicates that aging is possible in age groups from 15 to 69 years. Then people simply leave the analysis system (are removed from the analysis).

Constituent elements of formula 130 are based on the algorithms of the investigated submodel. The exception is the element "the change of occupation in the market module", which is based on the defined elements in the market equilibrium module. Therefore, the description of this element does not have the allocated thematic paragraph, the element formation is investigated in one equation, formula 131:

$$PMTB_{VgDPJEt} = \begin{cases} PMN_{VgDPJt} - PMDM_{VgDPJEt} - NPM_{VgDPJEt}, E = active \\ 0, E = non - active \end{cases}, \quad (131)$$

where

 $PMTB_{VgDPJE}$  - change of occupation in the market module;  $PMN_{VgDPJ}$  - employment after the change of occupation;  $PMDM_{VgDPJ}$  - change of occupation in order to get a job;  $NPM_{VgDPJ}$  - change of occupation of the employed.

Formula 131 shows that the occupation is changed only by economically active population. The change of analysis group of occupation in the labour analysis sub-model is caused by the change of occupation in order to get a job (the population is increased in groups with new occupations and, at the same time, is reduced in groups with old occupations, which is indicated by the difference between the elements "placement after change of occupation" and "change of occupation in order to obtain employment"), as well as the change of occupation by the employees

(which is summarily the change of occupation for employees). These elements are described more in the market equilibrium module. But the algorithms of labour analysis sub-model are described further.

The algorithms of admission for analysis and changes of population and structure are interrelated. Admission for analysis in a single population group causes the decline in population in other groups (except for the first-time admission, which is described below). Therefore, these two sub-algorithms are to be considered as a whole. At first the algorithms that are not related to the admission for the analysis (i.e., algorithms of mortality division, aging and removal from the analysis) are investigated, then the related algorithms are investigated and, finally, the algorithm related to the admission for analysis (algorithm of occupational supply).

# Algorithm of mortality division of working-age population by levels of education

Algorithm of mortality division of working-age population by levels of education is presented in Figure 2.37.



Figure 2.37. Algorithm of mortality division of working-age population by levels of education

The Figure 2.37 shows that in order to calculate the mortality in the group of levels of education, the population in the group is aggregated by age groups and gender dimensions. All groups of levels of education allow calculating the ratio of a particular group in the population. The ratio of the group in the population determines the ratio of mortality of the group in the total mortality (along with overall mortality, it allows to calculate how many people die in the group). The division of mortality people by occupations, fields of education and groups of economic activities is determined by the population structure.

The calculation of the population by age groups and gender is presented in formula 132:

$$I_{\text{VgDt}} = \sum_{i=P} \sum_{k=J} \sum_{l=E} I_{\text{VgDPJEt}}^{ikl} , \qquad (132)$$

where  $I_{VgD}$  - population by age groups and gender;  $I_{VgDPJE}$  - population; P - occupation; J - field of education; E - group of economic activity.

Calculation of population structure is presented in formula 133:

$$ISt_{\rm VgDPJEt} = \frac{I_{\rm VgDPJEt}}{I_{\rm VgDt}},$$
(133)

where  $ISt_{VgDPJE}$  - population structure;  $I_{VgDPJE}$  - population;  $I_{VgD}$  - population by age groups and gender.

Calculation of total population by age groups and gender is presented in formula 134:

$$KI_{\rm VgDt} = \sum_{i=L} I_{\rm VgDt}^{i} , \qquad (134)$$

where

 $KI_{VgD}$  - total population by age groups and gender;  $I_{VgD}$  - population by age groups and gender; L - group of level of education.

Calculation of the ratio of population by age groups and gender is presented in formula 135:

$$ISI_{\rm VgDt} = \frac{I_{\rm VgDt}}{KI_{\rm VgDt}},$$
(135)

where

 $ISI_{VgD}$  - population ratio of the total population by age groups and gender;  $I_{VgD}$  - population by age groups and gender;  $KI_{VgD}$  - total population by age groups and gender.

Mortality calculation is presented in formula 136:

$$M_{\rm VgDPJEt} = M_{\rm VgDt} \times ISI_{\rm VgDt} \times ISt_{\rm VgDPJEt}, \qquad (136)$$

where

 $M_{VgDPJE}$  - mortality;  $M_{VgD}$  - mortality by age groups and gender in 15-70-year-old groups;  $ISI_{VgD}$  - population ratio of the total population by age groups and gender;  $ISt_{VgDPJE}$  - population structure.

Formula 136 provides the mortality calculation by age groups, gender, occupations, fields of education and groups of economic activity. One equation includes the total mortality, group ratio of the total population and the internal structure of the population.

# Working-age population aging and removal from the analysis (retirement) algorithms divided by educational levels.

Working-age population aging algorithm divided by educational levels is largely repeating the mortality division of the population algorithm. They have the same function: to synchronize demographics and labour analysis sub-models. The algorithms use the same principle - calculation of the population structure and ratio, but differ by their application object (in the algorithm of mortality, i.e. mortality, but in the aging algorithm - aging). On the basis of this fact, the aging algorithm is not repeatedly described, but only one equation is reflected, which is different from these algorithms. It is the equation of aging calculation, formula 137:

$$NOV_{V_{\text{QDPJEt}}} = NOV_{V_{\text{QDT}}} \times ISI_{V_{\text{QDT}}} \times ISt_{V_{\text{QDPJEt}}},$$
(137)

where  $NOV_{VgDPJE}$  - aging;  $NOV_{VgD}$  - aging by age groups and gender in 15-70-year-old groups;  $ISI_{VgD}$  - population ratio of the total population by age groups and gender;  $ISt_{VgDPJE}$  - population structure.

Formula 137 shows the aging calculation by age, gender, occupations, fields of education, and groups of economic activity. One equation includes the total aging, the group ratio of the total population and the internal structure of the population.

Population division algorithm to remove from the analysis (retirement), as well as the aging division algorithm repeats the mortality division algorithm. Only one equation is different from them, which is presented in formula 138:

$$UN_{\rm VgDPJEt} = UN_{\rm VgDt} \times ISI_{\rm VgDt} \times ISt_{\rm VgDPJEt}, Vg = Vg70_74,$$
(138)

where

 $\label{eq:VgDPJE} \begin{array}{l} UN_{VgDPJE} \mbox{-} removal from analysis; \\ UN_{VgD} \mbox{-} removal from analysis by age groups and gender; \\ ISI_{VgD} \mbox{-} population ratio of the total population by age groups and gender; \\ ISt_{VgDPJE} \mbox{-} population structure. \end{array}$ 

Formula 138 presents the calculation of the removal from analysis by age groups, gender, occupations, fields of education, and groups of economic activity. In comparison with mortality or aging, removal from the analysis is applied only to one labour analysis age group 70 to 74 years, indicating that by reaching the age of 75, people leave the labour force and are removed from analysis.

## **Technical algorithm of education attainment**

Technical algorithm of education attainment is named as technical algorithm, as it more has a technical function - to move people from one education level analysis group to another education level at the moment of education attainment.

Technical algorithm of education attainment connects two education level analysis groups, so that the algorithm is necessary to be analysed, by estimating its impact on two education level analysis groups.

By the example of lower and secondary education groups the technical algorithm of education attainment is presented in Figure 2.38.



Figure 2.38. Technical algorithm of education attainment by the example of lower and secondary education groups

Figure 2.38 shows that by attaining education the population is admitted for analysis in the group according to the attained education. By acquiring education, the population obtains the occupation, which is indicated by the choice of occupation algorithm upon the education attainment. Admission of the population to the education level analysis group reduces the population in the analysis group of the previous educational level to the same extent by age groups and gender. But they have different (or may be different) occupations, fields of education, economic activity. The population of the certain age and gender is reduced in the analysis group of the previous educations, fields of education, economic activity are determined, taking into account the existing population in the group (selecting by the element "maximum of changes", as well as calculating its structure).

The calculation of the equation of admission for analysis is presented in formula 139:

$$UP_{V_{g}DPJEt} = II_{VJDt} \times PPS_{PJt} \times ULL_{V_{g}Dt}, \qquad (139)$$

where

UP<sub>VgDPJE</sub> - admission for analysis;

 $II_{VJD}$  - mastering of education at the level of general secondary and vocational studies;

 $PPS_{PJ}$  - structure of occupation demand in accordance with ECM for population with general secondary and vocational education;

 $ULL_{VgD}$  - present level of participation.

Formula 139 shows that the equation of admission for analysis adapts the education mastering and choice of occupation for the working age analysis algorithm. In accordance with formula 139, all people, acquiring new education, regardless of previous economic status, are divided by groups of economic activity, taking into account the provided level of participation by age groups and gender (participation level for the model is provided exogenously, it is described in details in the sub-model of labour economic activity). According to the attained education, the graduates obtain the occupation by entering the labour market. The occupation is obtained in accordance with the structure of the demand for occupations (from labour demand sub-model) and education compliance matrix (ECM).

By determining the decline in population in the group of previous level of education, the population selection is carried out for the calculation of the structure. The population selection is denoted in the model as a maximum of changes. The maximum of changes, i.e., the population forms the analysed age group and does not leave it at the next forecasting stage (year). Maximum change is necessary to prevent the developed algorithms simultaneously to modify the same

population groups more than it is allowed by the condition of analysis group. The calculation of the maximum of changes equation is presented in formula 140:

$$IZM_{\rm VgDPJEt} = \begin{cases} I_{\rm VgDPJEt} - M_{\rm VgDPJEt} - NOV_{\rm VgDPJEt}, Vg \in 15\_19...65\_69\\ I_{\rm VgDPJEt}, Vg = 70\_74 \end{cases},$$
(140)

where

$$\begin{split} & IZM_{VgDPJE} \text{ - maximum of changes;} \\ & I_{VgDPJE} \text{ - population;} \\ & M_{VgDPJE} \text{ - mortality;} \\ & NOV_{VgDPJE} \text{ - aging;} \\ & Vg \text{ - age group.} \end{split}$$

Formula 140 shows that the maximum of changes for age groups from 15 to 69 years is calculated as the difference between the population and the summed up mortality and aging. The maximum of changes of the older groups coincide with the population.

Maximum of changes structure is calculated in formula 141:

$$IZMSt_{\text{VgDPJEt}} = \begin{cases} \frac{IZM_{\text{VgDPJEt}}}{\sum_{k \in P} \sum_{n \in J} \sum_{m \in E} IZM_{\text{VgDPJEt}}^{knm}}, \frac{IZM_{\text{VgDPJEt}}}{\sum_{k \in P} \sum_{n \in J} \sum_{m \in E} IZM_{\text{VgDPJEt}}^{knm}} > 0\\ 0, \frac{IZM_{\text{VgDPJEt}}}{\sum_{k \in P} \sum_{n \in J} \sum_{m \in E} IZM_{\text{VgDPJEt}}^{knm}} \le 0\\ 0, \frac{1}{0} \end{cases}, \qquad (141)$$

where

 $IZMSt_{VgDPJE}$  - maximum of changes structure;  $IZM_{VgDPJE}$  - maximum of changes.

The maximum of changes structure is calculated by dividing the array elements by the total amount of these elements. In addition, formula 141 shows that the calculation of the maximum of changes structure is protected from situations, when there are no people in the array (i.e., it is not possible to calculate the structure). In this case, a value of zero is assigned to the corresponding structural elements. Population which abandon the previous education analysis group due to attainment of new education is calculated in formula 142:

$$VAGA_{V_{g}DPJEt} = IZMSt_{V_{g}DPJEt} \times \sum_{k \in P} \sum_{n \in J} \sum_{m \in E} UP_{V_{g}DPJEt}^{knm},$$
(142)

where  $VAGA_{VgDPJE}$  - old analysis group abandonment;  $IZMSt_{VgDPJE}$  - maximum of changes structure;  $UP_{VgDPJE}$  - admission for analysis.

Formula 142 reflects that the analysis group of the previous educational level is abandoned by the population of certain age and gender (the number coincides with the educated people by age and gender in the next level of education), but their occupations, fields of education, economic activity are determined in accordance with the population in the group and its structure. Abandonment of the old analysis group along with the results of other algorithms form the population and structural changes in the analysed education level group, formula 143:

$$ISSI_{\text{VgDPJEt}} = M_{\text{VgDPJEt}} + AM_{\text{VgDPJEt}} + VAGA_{\text{VgDPJEt}} + EM_{\text{VgDPJEt}} - IM_{\text{VgDPJEt}} - AMVGM_{\text{VgDPJEt}}, (143)$$

where  $ISSI_{VgDPJE} - change of population and structure;$   $M_{VgDPJE} - mortality;$   $AM_{VgDPJE} - change of the economic activity group of the population;$   $VAGA_{VgDPJE} - old analysis group abandonment;$   $EM_{VgDPJE} - emigration;$   $IM_{VgDPJE} - immigration;$   $MI_{VgDPJE} - maximum of changes;$   $AMVGM_{VgDPJE} - change of the economic activity group along with the change of age.$ 

Formula 143 reflects all parameters affecting the population and structural changes. Formula 143 reflects the last equation, which connects the various analysis groups of the education levels. Further the algorithm of occupational supply is described, which is related only to technical algorithm of education attainment.

## Algorithm of occupational supply

Algorithm of occupational supply provides obtainment of occupation upon education attainment, by entering the labour analysis group of the corresponding level. In the algorithm the occupation is given according to the attained education, occupation - according to education compliance matrix (ECM) and occupational demand. The algorithm of occupational supply is presented in Figure 2.39.



Figure 2.39. occupational supply development algorithm

Occupational supply structure is formed in accordance with the occupational demand structure. Occupational demand is determined in the occupational demand sub-model (formula 29). Multiplying the occupational demand by integrated educational compliance matrix (AECM, the development is analysed in formula 296), the occupational demand is determined according to the ECM. This indicates the required number of employees, whose occupation corresponds to the education. Then the structure of occupational demand according to ECM is determined.

The calculation of the occupational demand according to ECM is presented in formula 144:

$$PP_{\rm PJt} = \begin{cases} AIAM_{\rm PJt}, AIAM_{\rm PJt} \times LS_{\rm PJt} = 0\\ AIAM_{\rm PJt} \times LS_{\rm PJt}, AIAM_{\rm PJt} \times LS_{\rm PJt} \neq 0 \end{cases}$$
(144)

where

PP<sub>PJ</sub> - demand for occupation in accordance with ECM;

 $LS_{PJ}$  - labour demand by fields of education and occupations at the level of education; AIAM<sub>PJ</sub> - integrated education compliance matrix.

Occupational demand according to ECM is developed multiplying ECM by the occupational demand. In some cases the certain occupations may not be demanded. In this case, ECM determines the occupational demand. It is an artificial activity that allows to assign occupation to the education, which does not have a demand from the market side (model provides that the graduates begin their studies for the demanded occupations, but the demand during the study time may fall to zero, so the alumni acquire a non-demanded education and a non-demanded occupation).

Calculation of the occupational demand structure according to ECM is presented in formula 145:

$$PPS_{PJt} = \begin{cases} \frac{PP_{PJt}}{\sum_{l=P} PP_{PJt}^{l}}, \frac{PP_{PJt}}{\sum_{l=P} PP_{PJt}^{l}} > 0\\ 0, \frac{PP_{PJt}}{\sum_{l=P} PP_{PJt}^{l}} \le 0\\ 0, \frac{1}{2} 0, \frac{1}{2} 0 \end{cases},$$
(145)

where

 $PPS_{PJ}$  -demand structure of occupations in accordance with ECM;  $PP_{PJ}$  - demand for occupation in accordance with ECM; P - occupation; J - field of education.

In the equation of the occupational demand structure not only the structure is calculated, but the model is also protected from the situation when there is no demand in the specific field. In this case, a certain element of structure obtains the value of zero.

### Change of the economic activity group together with the change of age group

In statistical records the age groups differ in economic activity. In order to reflect the changes of economic activity in the model, along with the age changes, the labour analysis sub-model provides the algorithm of the economic activity group change along with the change of age group. This is presented in Figure 2.40.



Figure 2.40. Change of the economic activity group with the change of age group

Algorithm of the economic activity group change along with the change of age group is based on the aging parameter. Due to aging, the population, changing the age group, is determined. The number of analysis dimensions is reduced for this population and the groups of economic activity are summed up. Further, according to provided level of economic participation the economically active population is determined. The number of economically active population is compared with the aging result of the economically active population by determining the change of economic activity status of active population. These changes also cause the change of economically inactive population, which together form the change of the economic activity group along with the change of age groups.

Let us look at the constituent equations of this algorithm. Formula 146 reflects the calculation of the aging results:

$$NOVR_{VgDPJEt} = NOV_{(Vg-1)DPJEt} Vg \in \{Vg 20_24, ..., Vg 70_74\},$$
(146)

where  $NOVR_{VgDPJE}$  - aging result;  $NOV_{vgDPJE}$  - aging.

The aging result is the population, changing the age analysis group. It is equal to the population, leaving the previous age analysis group. Aging result in formula 147 has a reduced number of the analysis dimensions and economic activity groups are summed up:

$$NOVRb_{\rm VgDPJt} = \sum_{i \in E} NOVR_{\rm VgDPJEt}^{i} , \qquad (147)$$

 $NOVRb_{VgDPJ}$  - aging result without the economic activity in the groups;  $NOVR_{VgDPJE}$  - aging result.

The active population is calculated on the basis of the aging results without the groups of economic activity and provided participation level, formula 148:

$$ASP_{VgDPJt} = NOVRb_{VgDPJt} \times ULL_{VgDt}, \qquad (148)$$

where

 $ASP_{VgDPJ}$  - assignment of economic activity status to active population; NOVRb<sub>VgDPJ</sub> - aging result without the economic activity in the groups; ULL<sub>VgD</sub> - determined level of participation.

The calculated active population is compared with the aging results (the active population); the difference indicates to the change of the active population along with the change of age group, formula 149:

$$ASM_{VgDPJt} = NOVR_{VgDPJEt} - ASR_{VgDPJt}, E = active,$$
(149)

where

 $\begin{array}{l} ASM_{VgDPJ} \mbox{-} change \mbox{ of economic activity status for active population;} \\ NOVR_{VgDPJE} \mbox{-} aging result; \\ ASP_{VgDPJ} \mbox{-} assignment \mbox{ of economic activity status to active population.} \end{array}$ 

Changes in one group of economic activity cause the opposite changes in the second group of economic activity (i.e., by increasing the economically active population, the economically inactive population should be reduced). The summed up changes in the active and inactive population form the change of the economic activity group, formula 150:

$$AMVGM_{VgDPJEt} = \begin{cases} -ASM_{VgDPJt}, E = active\\ ASM_{VgDPJt}, E = non-active \end{cases}$$
(150)

where

 $AMVGM_{VgDPJE}$  - change of the economic activity group along with the change of age;  $ASM_{VgDPJ}$  - change of economic activity status for active population.

Formula 150 shows that the change of one economic activity group is inseparable from the other.

The unified algorithms reflected above form the base model. The features of certain algorithms are described below.

#### Features of unified algorithms in the labour analysis sub-model

The first feature of the algorithm is related to the initial admission for analysis of the population in the level of education groups. Unified models determine that the admission for analysis is possible by acquiring education. So, by acquiring basic education and at the same time reaching the age of 15, people are admitted for analysis. Unified algorithms do not provide that any

part of the population may not get education. Formula 151 shows the feature of the lower education level, which provides the admission of 15-year-old population for analysis without education:

$$UP_{\rm VgDPJEt} = (II_{\rm VJDt} + BI_{\rm VJDt}) \times PPS_{\rm PJt} \times ULL_{\rm VgDt}, \qquad (151)$$

where

 $\begin{array}{l} UP_{VgDPJE} \mbox{-} admission for analysis; \\ II_{VJD} \mbox{-} acquisition of education; \\ BI_{VJD} \mbox{-} 15\mbox{-} year\mbox{-} old population without education; \\ PPS_{PJ} \mbox{-} demand structure of occupations in accordance with ECM; \\ ULL_{VgD} \mbox{-} provided level of participation. \end{array}$ 

Comparing the algorithm modification (formula 151) with a unified algorithm (formula 139), it is evident that the population without education is added to the population with the lowest education level.

The calculation of uneducated 15-year-old people is presented in formula 152.

$$BI_{\rm Dt} = I_{\rm VDt} - \sum_{i=J} II_{\rm VJDt}^{i}, V = 15, \qquad (152)$$

where

 $BI_D - 15$ -year-old population without education;  $I_{VD}$  - population;  $II_{VJD}$  - acquisition of education; J - field of education.

15-year-old population without education is calculated as the 15-year-old population minus the 15-year-old population with education (basic education in the age of 15). In order to apply the model, the number of the 15-year-old population without education is included in the age group from 15 to 19 with unclassified field of education.

The other feature of the algorithm is related to the analysis of the population with a doctoral education. In this group the population cannot improve their education and accordingly it cannot be changed, taking into account the next level of education. The algorithm modifies the population and structural changes, formula 153:

$$ISSI_{\rm VgDPJEt} = M_{\rm VgDPJEt} + AM_{\rm VgDPJEt} + EM_{\rm VgDPJEt} - IM_{\rm VgDPJEt} - AMVGM_{\rm VgDPJEt}, \qquad (153)$$

where

$$\begin{split} & ISSI_{VgDPJE} \text{ - change of population and structure;} \\ & M_{VgDPJE} \text{ - mortality;} \\ & AM_{VgDPJE} \text{ - change of economic activity group of population;} \\ & EM_{VgDPJE} \text{ - emigration;} \\ & IM_{VgDPJE} \text{ - immigration;} \\ & AMVGM_{VgDPJE} \text{ - change of economic activity group along with the change of age.} \end{split}$$

Comparing the equation of the population analysis with doctoral education (formula 153) to the unified equation (formula 143), it is evident that the element of "old analysis group abandonment" is taken away from the equation in the doctoral level group, which is related to the movement to another education analysis group.

## 2.2.4. Labour force participation sub-model

Labour force participation sub-model is based on the assumption that at the forecasting moment the labour economic activity is known, i.e., the labour participation in the labour market by age group and gender is known, which is an exogenously provided parameter. According to this assumption, the task of labour force participation sub-model is to simulate labour economic activity by levels, fields of education, gender, occupations and age groups from exogenously provided labour economic activity by age groups and gender.

In order to implement this, the model provides two algorithms: the first divides the population by economic activity in accordance with educational levels, and, the second, which is unified and used at every level of education, divides the population by economic activity in accordance with fields of education and occupations. Visually it is presented in Figure 2.41.



Figure 2.41. Algorithms of labour force participation sub-model

The Figure 2.41 shows that the labour in economic activity sub-model consists of two algorithms. They will be examined in more detail further.

# Algorithm of division of target economically active population by levels of education

This algorithm is not unified and is used only once, dividing the economically active population by levels of education. The nature of the algorithm is described in Figure 2.42.



Figure 2.42. Algorithm of division of target economically active population by levels of education

The algorithm consists of two parts: primarily, the provided participation level (ratio of economically active population by age and gender) and, secondly, the population. Multiplying the provided participation level (exogenous parameter) by the labour amount (total population by age groups and gender (working age), from the demographic sub-model), the provided (target) number of economically active population (by age groups and gender) is obtained.

At the same time, knowing the economically active population by levels of education, age groups and gender, it is possible to calculate the structure. This structure is not the target structure, but it is the structure of real condition. On this basis, the target division by educational levels (assuming that the structure of economic activity at the levels of education in the future will remain the same) is calculated.

The target economically active population by educational levels, age groups and gender is calculated in accordance with the provided economically active population by age groups and gender, and the real structure by educational levels. According to their calculation, for each level of education appropriate target economically active population is selected (by age groups and gender).

Let us investigate the forming equations of the algorithm.

Provided economically active population is calculated in formula 154:

$$UEAI_{VeDt} = ULL_{VeD} \times KI_{VeDt}, \qquad (154)$$

where

 $\ensuremath{\mathsf{UEAI}_{\mathsf{VgD}}}\xspace$  - provided economically active population by age groups and gender;

ULL<sub>VgD</sub> - provided level of participation;

 $KI_{\text{VgD}}$  - total population by age groups and gender.

On the basis of the provided economically active population (by age groups and gender) the number of target economically active population by educational levels, age groups and gender is calculated (formula 155):

$$MEAI_{VgDLt} = UEAI_{VgDt} \times EAISt_{VgDLt}, \qquad (155)$$

where

 $MEAI_{VgDL}\xspace$  - target economically active population by levels of education, age groups and gender;

UEAI<sub>VgD</sub> - provided economically active population by age groups and gender;

 $EAISt_{VgDL}$  - structure of economically active population by levels of education, age groups and gender.

For calculation of the target economically active population the provided economically active population (from formula 154) and the structure of the economically active population, presented in formula 156, are used:

$$EAISt_{VgDLt} = \begin{cases} \frac{EAI_{VgDLt}}{\sum_{i \in L} EAI_{VgDLt}^{i}}, \frac{EAI_{VgDLt}}{\sum_{i \in L} EAI_{VgDLt}^{i}} > 0\\ 0, \frac{EAI_{VgDLt}}{\sum_{i \in L} EAI_{VgDLt}^{i}} \le 0\\ 0, \frac{1}{0} \end{cases},$$
(156)

where

 $EAISt_{VgDL}$  - structure of economically active population by levels of education, age groups and gender;

 $EAI_{VgDL}$  - number of economically active population by levels of education, age groups and gender.

In addition to the calculation of structure, formula 156 presents the model and data technical features: the structure is calculated only if mathematically possible, but if it is impossible to calculate the structure, all elements are assigned the value of zero.

The economically active population, presented in formula 156, is calculated on the basis of labour analysis algorithm, but its forming equation is included in economic activity sub-model, as it is not used anywhere else. The calculation of economically active population is presented in formula 157:

$$EAI_{VgDLt} = \left[ \left[ \sum_{n \in P} \sum_{k \in J} I_{VgDPJE_{-1}t}^{nk} \right]_{1} \cup \left[ \sum_{i \in P} \sum_{k \in J} I_{VgDPJE_{-2}t}^{nk} \right]_{2} \cup \dots \cup \left[ \sum_{i \in P} \sum_{k \in J} I_{VgDPJE_{-}(i-1)t}^{nk} \right]_{i-1} \cup \left[ \sum_{i \in P} \sum_{k \in J} I_{VgDPJE_{-1}t}^{nk} \right]_{i} \right]_{i} = active, \quad (157)$$

where

 $EAI_{VgDL}$  - number of economically active population by levels of education, age groups and gender;

 $I_{VgDPJE}$  - population.

Formula 157 shows that the economically active population by educational level is calculated by aggregating the economically active population of all educational levels, reducing the number of analysis dimensions, i.e., summing up the field of education and occupation groups.

Formula 157 describes the last equation of the analysed algorithm. Further the algorithm of change of economic activity group of the population is described.

# Algorithm of change of target economic activity group of the population

Algorithm of change of economic activity group of population is unified, which can be used for each level of education. The nature of the algorithm is described in Figure 2.43.



Figure 2.43. Algorithm of change of target economic activity group of the population

Algorithm of change of economic activity group of the population in the beginning reminds the algorithm of division of economic activity by educational levels: the target economically active population is calculated from provided population and the actual economic population structure. The difference is in dimensions: if previously the level of education was analysed, now the structure involves the fields of education and occupations.

Upon determination of the target economically active population, it is necessary to calculate the changes of the population to achieve the target level. Changes are calculated in such a way that

the difference between the current and target participation level is included in the parameter "change of the economic activity groups of the population in accordance with the target".

If it is necessary to increase the economically active population, this parameter is positive (greater than zero), and this part is reflected by the parameter "positive change of the economic activity groups of the population in accordance with the target". If it is necessary to reduce the economically active population, this parameter is negative (less than zero), and this part is reflected by the parameter "negative change of the economic activity groups of the population in accordance with the target", as indicated by Figure 2.37.

Changes in one economic activity group (in our case, the active population) give rise to the opposite changes in the second economic activity group (i.e., by increasing the economically active population, it is necessary to reduce the economically inactive population). Therefore, the positive change of economic activity group affects both the growth of the active population and the growth of inactive population (but in this case it would be a minus sign, i.e. decline). The same can be related to the negative change of economic activity group.

By summing up the growth (or decline) of active and inactive population, the total change of economic activity of population is calculated.

Let us investigate the forming equations of the algorithm.

Primarily, the unified algorithm for level of education begins selecting the economically active population of the relevant level from the target economically active population by educational levels, formula 158:

$$MEAI_{V_gDt} = MEAI_{V_gDLt}, L = i,$$
(158)

where

MEAI<sub>VgD</sub> - target economically active population by age groups and gender at *i* level of education;

 $MEAI_{VgDL}$  - target economically active population by level of education, age groups and gender;

L - level of education.

The target economically active population by age groups and gender is further divided by occupation and education groups, formula 159.

$$MEAI_{V_gDPJ_t} = \left(MEAI_{V_gD_t} \times EAISt_{V_gDPJ_t}\right) \wedge \sum_{k \in E} I^k_{V_gDPJ_{Et}}, \qquad (159)$$

where

MEAI<sub>VgDPJ</sub> - target economically active population;

 $MEAI_{VgD}$  - target economically active population by age groups and gender;

 $EAISt_{VgDPJ}$  - structure of economically active population by age groups, gender, occupations and fields of education;

 $I_{VgDPJE}$  - population.

Calculation of the target economically active population involves the target economically active population by age groups and gender, and the population structure by age groups, gender, occupations and fields of education. In addition to formula 159, each target economically active population group is compared to the total population (by summing up the economically active and inactive population); in the result the lowest index is selected to prevent exceeding of the target economically active population over the total population. Population is calculated in the labour analysis algorithm, but the structure of the economically active population is calculated in formula 160.

$$EAIS_{k_{g}DPJt} = \begin{cases} \frac{I_{V_{g}DPJEt}}{\sum_{k \in J} \sum_{i \in P} I_{V_{g}DPJEt}^{k_{i}}}, \frac{I_{V_{g}DPJEt}}{\sum_{k \in J} \sum_{i \in P} I_{V_{g}DPJEt}^{k_{i}}} > 0\\ 0, \frac{I_{V_{g}DPJEt}}{\sum_{k \in J} \sum_{i \in P} I_{V_{g}DPJEt}^{k_{i}}} \le 0 \qquad , E = active\\ 0, \frac{1}{0} \end{cases}$$
(160)

EAISt<sub>VgDPJ</sub> - structure of economically active population by age groups, gender, occupations and fields of education;

 $I_{VgDPJE}$  - population.

In addition to the structure calculation formula 160 reflects the model and data technical features: the structure is calculated only if mathematically possible, but if it is impossible to calculate the structure, all elements are assigned the value of zero.

Upon determination of the target economically active population, it is compared to the current active population, defining the necessary changes to achieve the target level, formula 161:

$$EAIM_{V_gDPJt} = MEAI_{V_gDPJt} - EAI_{V_gDPJt} = MEAI_{V_gDPJt} - I_{V_gDPJt}, E = active,$$
(161)

where

EAIM<sub>V $\circ$ DPI</sub> - change of economic activity group of the population in accordance with the target;

MEAI<sub>VgDPJ</sub> - target economically active population; EAI<sub>VgDPJ</sub> - economically active population;  $I_{VgDPJE}$  - population.

The change of economic activity group of the population is calculated in two ways: the economically active population is subtracted from the target economically active population or the total population selecting only active population, which is presented in formula 161.

The change of economic activity group provides positive (greater than zero) and negative (less than zero) changes. In this case, positive changes indicate the growth of economically active population (at the same time decline in economically inactive population), while the negative changes indicate the decline in economically active population (at the same time growth of the economically inactive population). The division of changes into positive and negative is presented in formulas 162 and 163:

$$PEAIM_{V_gDPJ_t} = EAIM_{V_gDPJ_t} \lor 0, \qquad (162)$$

where

PEAIM<sub>VgDPJ</sub> - positive change of economic activity group of the population in accordance with the target;

 $EAIM_{V_{SDPI}}$  - change of economic activity group of the population in accordance with the target;

$$NEAIM_{V_gDPJ_t} = -(EAIM_{V_gDPJ_t} \land 0), \tag{163}$$

 $NEAIM_{VgDPJ}$  - negative change of economic activity group of the population in accordance with the target;

 $EAIM_{VgDPJ}$  - change of economic activity group of the population in accordance with the target;

Positive and negative changes of the economic activity group are calculated comparing the change of the group to zero. In addition, the negative change of economic group receives another sign, i.e. negative (less than zero) number receives the positive (greater than zero) number. In the result the further calculated index "growth of inactive population" is positive.

Growth of economically active and inactive population is presented in formulas 164 and 165:

$$EAIP_{V_gDPJ_t} = PEAIM_{V_gDPJ_t} - NEAIM_{V_gDPJ_t}.$$
(164)

where

EAIP<sub>VgDPJ</sub> - growth of active population;

 $\text{PEAIM}_{\text{VgDPJ}}$  - positive change of economic activity group of the population in accordance with the target;

 $NEAIM_{VgDPJ}$  - negative change of economic activity group of the population in accordance with the target.

$$ENIP_{V_gDPJ_t} = NEAIM_{V_gDPJ_t} - PEAIM_{V_gDPJ_t}.$$
(165)

where

 $ENIP_{VgDPJ}$  - growth of inactive population;

 $NEAIM_{VgDPJ}$  - negative change of economic activity group of the population in accordance with the target;

 $PEAIM_{VgDPJ}$  - positive change of economic activity group of the population in accordance with the target.

Calculating the growth of economically active and inactive population, positive and negative changes of economic activity groups are taken into account twice: primarily, they increase the relevant population group, secondly - reduce the population of the opposite group (i.e., with the growth of economically active population, economically inactive population declines, and vice versa). Taking into account the equation specifics, the growth of economically active and inactive population can be both positive and negative, that is, they reflect both population growth and decline in the groups.

Knowing the economically active and inactive population growth, it is possible to calculate the resulting change of economic activity group of population, formula 166:

$$AM_{V_gDPJEt} = \left[ \left[ EAIP_{V_gDPJt} \right] \cup \left[ ENIP_{V_gDPJt} \right] \right].$$
(166)

where

 $AM_{VgDPJE}$  - change of economic activity group of the population;

 $EAIP_{VgDPJ}$  - growth of active population;

 $ENIP_{VgDPJ}$  - growth of inactive population.

The change of economic activity group of population integrates the growth of economically active and inactive population into a single matrix, giving the status of the economic activity group to the elements of the group.

# 2.2.5. International migration sub-model

International migration sub-model uses unified algorithms according to analysis groups of the population education level. Overall scheme of the unified algorithm of the international migration is presented in Figure 2.44.



# Figure 2.44. Overall scheme of the unified algorithm of the international migration

Figure 2.44 shows that the international migration algorithm of the population is divided into two parts: emigration and immigration. Two different algorithms are applied to those parts.

Economic migration for both algorithms is formed by the salary ratio. Two pairs of salaries are used to compare the salary: "target salary ratio", which evaluates the relationship of the salary

level between two highly developed countries, at which the economic migration has a random nature, and "salary ratio", which compares the Latvian and EU average salaries. The closer these indexes are and the less the difference is, the less the economic migration cause is. This is presented by economic migration (emigration and immigration) formation indexes in Figure 2.44.

Emigration is determined by the economic emigration formation index, emigration propensity (which is a fixed factor) and the population in the analysis group, it is represented by the element "labour emigration according to the current structure". While forecasting the labour market development, emigration amount is partly simulated and partly controllable parameter, so the labour emigration does not take place in accordance with the current labour structure. The emigration in the model takes place according to the provided economic activity level. This means that the element "labour emigration according to the current structure" determines labour emigration amounts by occupations, gender, age and education groups, while the next element "labour emigration according to the provided structure" determines the labour migration by economic activity levels. The element "total emigration" compares emigration and population if necessary, reducing the emigration in order to prevent the emigration amounts to exceed the population.

Immigration in sub-model is formed of three aspects: return immigration, economic migration and demanded immigration.

The significant proportion of immigration is formed by returned emigrants, called as the "return migration" in algorithm. It depends on the improvement of economic situation in the country, in particular - on the changes of long-term vacant working places (i.e., growth). With the increase of the number of long term vacant working places, the return immigration increases and may be equal to emigration amounts.

Economic immigration consists of long-term vacant working places in the country, economic immigration index (as a measure of the salary level), but it is limited by the readiness of the local employers to invite the employees from abroad, as well as the time required for foreigners to integrate into Latvian society.

Additionally, immigration covers potential long-term labour shortage in the country. If the GDP growth is projected, which cannot be ensured by the increase in productivity, and there are no labour reserves in the country, the immigration can solve labour shortage problem. This immigration is called as the demanded immigration in algorithm. It mainly depends on the long-term vacant working places and their dynamics. The developed algorithm allows determining the level of the demanded immigration impact on the labour market, i.e. modelling of the state migration policy outcomes (the elements "immigration ratio for solution of the labour shortage problem" and "method of definition of migration types" are integrated in the user interface and allow modelling different scenarios).

Summing up economic return and the demanded immigration, the labour immigration is obtained according to the current structure. Forecasting labour market development, immigration amount is partly simulated and partly controllable parameter, so that labour migration does not take place in accordance with the current immigration structure determined by the market. The immigration in the model takes place according to the provided economic activity level. It means that the element "labour immigration in accordance with the current structure" determines the labour immigration levels by occupation, gender, age and education groups, but the next element "total immigration" determines the labour migration by economic activity levels. Unlike emigration, immigration amounts are not dependent on the population in the analysed group.

Investigation of the constituent equations of the international migration sub-model begins with the investigation of the emigration equations. Formula 167 presents the investigation of the economic emigration formation index:

$$EEVK_{t} = (MAAK - AAK_{t}) \lor 0, \qquad (167)$$

where EEVK - economic emigration formation index; MAAK - target salary ratio; AAK - salary ratio.

Economic emigration formation index compares two pairs of salaries "target salary ratio", which evaluates the relationship of the salary level between two highly developed countries, at which the economic migration has the random nature, and "salary ratio", which compares the Latvian and EU average salaries. The closer these indexes are and the less the difference is, the less the economic migration cause is. Economic emigration formation index is always greater than zero, i.e., with the increase of the salary level in the country up to EU level or more, the index decreases only to zero, and emigration does not become an immigration.

Economic emigration formation index allows calculating the labour emigration amounts according to current labour structure, which is presented in formula 168:

$$EMES_{\rm VgDPJEt} = \frac{I_{\rm VgDPJEt} \times ET}{TML} \times EEVK_{t}, \qquad (168)$$

where

 $EMES_{VgDPJE}$  - labour emigration in accordance with the current structure;

I<sub>VgDPJE</sub> - population;

ET - emigration propensity;

TML - time to take the emigration decision;

EEVK - economic migration formation index.

Formula 168 shows that emigration depends on the population in the group, emigration propensity, time of decision making on emigration and economic emigration formation index.

In accordance with the structure in formula 169, the labour emigration has been changed according to the provided economic activity level:

$$EMUS_{VgDPJEt} = \left(\sum_{i \in E} EMES_{VgDPJEt}^{i}\right) \times \begin{cases} ULL_{VgDt}, E = active\\ (1 - ULL_{VgDt}), E = non - active \end{cases}$$
(169)

where

 $EMUS_{VgDPJE}$  - labour emigration in accordance with the provided structure;  $EMES_{VgDPJE}$  - labour emigration in accordance with the current structure;  $ULL_{VgD}$  - provided participation level.

Formula 169 shows that according to the current structure, the economic activity groups are summed up for the labour emigration, and then the new economic activity groups are assigned according to the provided participation level.

On the basis of the labour emigration the total emigration is calculated according to the current structure, formula 170:

$$EM_{V_{g}DPJEt}^{l} = EMUS_{V_{g}DPJEt} \wedge I_{V_{g}DPJEt} , \qquad (170)$$

 $EM_{VgDPJE}^{1}$  - total emigrations at the education level 1;  $EMUS_{VgDPJE}$  - labour emigration in accordance with the provided structure;  $I_{VgDPJE}$  - population.

The resulting emigration is calculated as the minimum number of people in the calculated labour emigration according to the provided structure and in the analysed labour group. This prevents more people to emigrate than there are in the system.

Unlike emigration, immigration has several causes. Model analyses three causes of immigration: return immigration, economic immigration and demanded immigration. The return immigration is analysed first.

Return immigration is related to the return of emigrants. It depends on the improvement of economic situation in the country, in particular - with the changes of long-term vacant working places (i.e., growth). Return immigration is calculated in formula 171:

$$AIM_{V_{g}DPIEt} = EM_{V_{g}DPIEt} \times (((1 - INDVI_{t}) \lor 0) \land 1),$$
(171)

where

 $\label{eq:starses} \begin{array}{l} AIM_{VgDPJE} \mbox{-} return immigration; \\ EM_{VgDPJE} \mbox{-} emigration; \\ INDVI \mbox{-} changes of long-term vacant working places. \end{array}$ 

Formula 171 shows that the return of emigrants is dependent on the changes of long-term vacant working places, which determine the ratio of the return of emigrants. As formula 170 shows, the ratio of the return of emigrants is limited from 0 to 1, as required by the technical realization of the model. Model predicts - if the economic situation gets worse, the return immigration may be reduced up to zero.

The changes of long-term vacant working places from formula 171 are calculated in formula 172:

$$INDVI_{t} = \begin{cases} \frac{INDV_{(t-1)} - INDV_{t}}{INDV_{(t-1)}}, INDV_{(t-1)} > 0 \\ 0; INDV_{(t-1)} \le 0 \end{cases} = \begin{cases} \frac{IINDV_{t} - INDV_{t}}{IINDV_{t}}, IINDV_{t} > 0 \\ 0; IINDV_{t} \le 0 \end{cases},$$
(172)

where

INDVI - changes of long-term vacant working places;

INDV - long-term vacant working places;

IINDV - long-term vacant working places of the previous period.

The changes of long-term vacant working places can be calculated in two ways: directly from long-term vacant working places and by introducing the intermediate parameter - long-term vacant working places in the previous period. Regardless of the type of calculation, the parameter reflects the relative changes of long-term vacant working places. This calculation requires the long-term vacant working places, described in formula 173:

$$INDV_{t} = \frac{\sum_{k \in P} \sum_{i \in D} \sum_{m \in J} VS_{DPJt}^{kim}}{INDVD}, \qquad (173)$$

where INDV - long-term vacant working places; VS<sub>DPJ</sub> - number of vacancies; INDVD - definition of long-term vacant working places.

In the calculation of the long-term vacant working places the number of vacancies and ratio index of the long-term vacant working places (definition) have been used, which present the time when the vacancies become long-term vacancies. Balancing module subchapter describes creation of vacancies in detail.

The second type of migration is economic immigration. Calculation of economic immigration is similar to the calculation of economic emigration. It is based on the economic immigration formation index, which evaluates salaries in several countries. The calculation of the economic immigration formation index is presented in formula 174:

$$EIVK_{t} = (AAK_{t} - MAAK) \lor 0, \qquad (174)$$

where

EIVK - economic migration formation index; AAK - salary ratio; MAAK - target salary ratio.

The calculation of the economic immigration formation index (formula 174) is similar to the calculation of the economic emigration formation index (formula 167). Comparing these indexes we can see that in one case the difference in salaries is used to evaluate the economic benefits for emigration, but in the second case - for immigration.

The calculation of the economic immigration is presented in formula 175:

$$EIM_{VgDPJEt} = \frac{INDV_{t} \times GAAD \times EIVK_{t}}{TILS} \times IMSt_{Vg} \times INDVSt_{DPJt}, E \in Active,$$
(175)

where

$$\begin{split} EIM_{VgDPJE} &- economic immigration; \\ INDV - long-term vacant working places; \\ GAAD - readiness to invite the foreign employees; \\ EIVK - economic migration formation index; \\ TILS - time to get integrated into Latvian society; \\ IMSt_{Vg} - age structure of immigrants; \\ INDVSt_{DPJ} - structure of long-term vacant working places; \\ E - economic activity group. \end{split}$$

Economic immigration amounts determine the long-term vacant working places, but they reduce the index "readiness to invite the foreign employees", economic immigration formation index and the time necessary for integration into Latvian society. The economic immigration structure is determined by the age structure and the structure of longstanding unoccupied working places, as indicated by formula 175. The calculation of the economic immigration uses constant indexes that can be changed when planning the national policy on immigration.

In formula 175 all elements have been examined apart from the element "structure of long-term vacant working places", which is calculated in formula 176:

$$INDVSt_{t} = \begin{cases} \left(\frac{VS_{DPJt}}{INDVD}\right), \left(\frac{VS_{DPJt}}{INDVD}\right), \frac{(VS_{DPJt})}{INDV} > 0\\ 0, \frac{(VS_{DPJt})}{INDV} \le 0\\ 0, \frac{(VS_{DPJt})}{INDV} \le 0 \end{cases}$$
(176)

 $INDVSt_{DPJ}$  - structure of long-term vacant working places;

 $VS_{DPJ}$  - number of vacancies;

INDVD - definition of long-term vacant working places;

INDV - long-term vacant working places.

The structure of long-term vacant working places is determined by dividing the long-term vacant working places by gender, occupations and fields of education by total number of long-term vacant working places. The number of long-term vacant working places by gender, occupations and fields of education are calculated from the number of vacancies and definition of long-term vacant working places (this is explained in formula 173). Additionally, formula 176 reflects that when it is not possible to calculate the structure of long-term vacant working places, the relevant structure elements are given the value of zero.

Further the required immigration has been investigated. It should be determined, based on the state global and long-term labour shortage.

In the demanded immigration the formation of vacancies (long-term vacancies) does not immediately create the immigration flow. These vacancies may be terminated by changing the occupations of the employees or by means of education systems. It means that the required immigration is caused by vacancies, which have a status of vacant working places for a long time, as marked in the model "long-term vacant working places, which form immigration". This calculation is presented in formula 177:

$$INDVM_{t} = (INDV_{t} \land INDV_{(t-TVSM)}) \times IMITPR = (INDV_{t} \land OIINDV_{t}) \times IMITPR, \qquad (177)$$

where

INDVM - long-term vacant working places, which are forming immigration;

INDV - long-term vacant working places;

TVSM - time for occupation of vacancies for migration;

OIINDV - long-term vacant working places of the second level in the previous period;

IMITPR - immigration ratio in the solution of the labour shortage problem.

The long-term vacant working places, which form immigration, can be calculated in two ways: directly from long-term vacant working places and by introducing the intermediate parameter - long-term vacant working places of the second level in the previous period. Formula 177 shows that the migration consists of a number of vacancies which have not been reduced in a certain time (time for occupation of vacancies for migration). In addition, formula 177 specifies the national policy planning element - immigration ratio in the solution of the labour shortage problem, which is dependent on the choice of the model user and is integrated into the user management interface of the model.

The demanded immigration calculation is based on the migration forming working places, formula 178:

(178)

where

 $\label{eq:pi_gdp_jd} \begin{array}{l} \text{PIM}_{VgDPJE} \text{ - demanded immigration;} \\ \text{INDVM - long-term vacant working places, which form immigration;} \\ \text{IMSt}_{Vg} \text{ - age structure of immigrants;} \\ \text{INDVSt}_{DPJ} \text{ - structure of long-term vacant working places;} \\ \text{IMVNV - method to define the immigration types;} \\ \text{E - economic activity group.} \end{array}$ 

The requested immigration amount is determined by long-term vacant working places, which are forming immigration. The structure of the required immigration is determined by the age structure of immigrants and structure of long-term vacant working places. Additionally, formula 178 reflects the national policy planning element "method to define immigration type", which excludes or switches off the required immigration calculation, depending on the user management interface of the model.

Summing up the return, economic and required immigration, labour immigration is calculated in accordance with the current structure, formula 179:

$$IMES_{V_{g}DPJEt} = AIM_{V_{g}DPJEt} + EIM_{V_{g}DPJEt} + PIM_{V_{g}DPJEt},$$
(179)

where

$$\begin{split} IMES_{VgDPJE} \mbox{-} labour immigration in accordance with the current structure;} \\ AIM_{VgDPJE} \mbox{-} return immigration;} \\ EIM_{VgDPJE} \mbox{-} economic immigration;} \\ PIM_{VgDPJE} \mbox{-} demanded immigration.} \end{split}$$

Labour immigration in accordance with the current structure does not ensure the sufficient controllability of the model results. To ensure the controllability options of the model results, the immigration according to the current structure is converted according to the provided structure, by calculating the resulting immigration, formula 180:

$$IM_{VgDPJEt}^{l} = \left(\sum_{i \in E} IMES_{VgDPJEt}^{i}\right) \times \begin{cases} ULL_{VgDt}, E = active\\ (1 - ULL_{VgDt}), E = non - active, \end{cases}$$
(180)

where

 $IM_{VgDPJE}^{l}$  - total immigrations at the education level 1; IMES<sub>VgDPJE</sub> - labour immigration in accordance with the current structure; ULL<sub>VgD</sub> - provided participation level.

Formula 180 shows that for the labour immigration, according to the current structure, the economic activity groups are summed up, and the new economic activity groups are defined according to the provided participation level, obtaining the resulting immigration.

Formula 180 presents the last equation of international migration sub-model unified algorithm of the educational levels. Sub-model also contains non-unified equations. They ensure international migration, labour analysis and demographic sub-model synchronization. This is presented in Figure 2.45.



Figure 2.45. International migration, labour analysis and demographic sub-model synchronization

The unified algorithms of the international migration sub-model calculate emigration and immigration by educational levels, which directly without processing are used in labour analysis sub-model in the unified algorithms by educational levels. To synchronize demography and international migration sub-models from unified algorithms, the total emigration and immigration are calculated by levels of education. Further, provided that the international migration sub-model operates with 5-year age groups, but the demographic sub-model analyses the population by one-year age groups, migrants from 5-year age groups are divided by one-year age groups.

Summed up emigration and immigration calculation is reflected in formulas 181 and 182:

$$EM_{\rm VgDPJEt} = \sum_{i \in L} EM_{\rm VgDPJEt}^{\rm li}, \qquad (181)$$

where

 $EM_{VgDPJE}$  - total emigration;

 $EM_{VgDPJE}^{1}$  - total emigration at the education level 1.

$$IM_{\rm VgDPJEt} = \sum_{i \in L} IM_{\rm VgDPJEt}^{\rm li} , \qquad (182)$$

where

IM<sub>VgDPJE</sub> - total immigration;

 $IM_{VgDPJE}^{l}$  - total immigration at the education level 1.

The total migration is calculated as the sum of the migration by educational levels.

Migration conversion from five-year age groups by one-year age groups is presented in formulas 183 and 184:

$$EM_{VDt} = \begin{cases} \sum_{n \in E} \sum_{k \in J} \sum_{i \in P} EM_{VgDPJEt}^{nki}, 15 \le V < 19, Vg = Vg15\_19\\ 5 & ...\\ \sum_{n \in E} \sum_{k \in J} \sum_{i \in P} EM_{VgDPJEt}^{nki}, 70 \le V < 74, Vg = Vg70\_74 \end{cases}$$
(183)

where

 $EM_{VD}$  - emigration by age groups and gender;  $EM_{VgDPJE}$  - summed up emigration.

$$IM_{VDt} = \begin{cases} \sum_{n \in E} \sum_{k \in J} \sum_{i \in P} IM_{VgDPJEt}^{nki}, 15 \le V < 19, Vg = Vg15\_19\\ \dots\\ \sum_{n \in E} \sum_{k \in J} \sum_{i \in P} IM_{VgDPJEt}^{nki}, 70 \le V < 74, Vg = Vg70\_74 \end{cases}$$
(184)

 $IM_{VD}$  - immigration by age groups and gender;  $IM_{VgDPJE}$  - summed up immigration.

Formulas 183 and 184 reflect the fact that the international migration and demographic submodel synchronization requires the reduction of the number of migration dimensions (population is summed up by occupations, fields of education and economic activity groups), and the population is divided from the five-year age groups in the equal proportion by the relevant one-year age groups.

# 2.3. Labour market module: balancing labour demand and supply



Structure of labour market module is presented in Figure 2.46.

Figure 2.46. Structure of labour market module and relation to other model modules

The Figure 2.46 shows that the labour market module consists of two sub-models: labour market and wage sub-models. Labour market sub-model combines labour demand and supply, taking into account the labour amounts and wages. Formation of labour supply and demand has been analysed above. Wage sub-model analyses the most important processes (changes of productivity) in the national economy and labour demand and supply amounts, resulting in the wages. Labour wages affect both labour market balance and labour supply formation, that is, the choice of education and international migration.

# 2.3.1. Labour market sub-model

SDM model in the labour market sub-model combines the labour supply and demand by determining labour employment, number of jobseekers, number of working places and number of occupied working places. On the basis of these key elements such important parameters as the number of vacancies, etc. are calculated. The scheme of the labour market sub-model algorithms is presented in Figure 2.47 (next page).

Figure 2.47 presents the labour market sub-model stocks, flows, key elements and algorithms scheme. Figure 2.74 reflects all stocks and flows, the impact of the algorithm on them, but it does not present the impact of the stocks and flows on key elements of the algorithms in detail. Figure 2.47 provides presentation of the operation logic and stock - flow system of the module. The logical structure of the sub-model is analysed in detail in the next subsection.

# 2.3.1.1. Framework and structure of the labour demand and supply balancing sub-model

SDM model provides analysis of the labour market by educational levels, each of the educational levels uses the unified labour balancing sub-model, which is presented in Figure 2.47. Combining labour balancing sub-models by levels of education, the overall labour market sub-model is presented in the following way, Figure 2.48 (next page).


Figure 2.47. Scheme of the algorithm of the labour market sub-model



Figure 2.48. Labour market sub-model

Summing up the results of unified labour market sub-models by educational levels, the key model elements have been calculated: total number of jobseekers, total number of employed, total number of vacancies. On this basis, the key labour market indicators have been calculated: the number of jobseekers by educational levels, the number of jobseekers by gender, jobseekers by occupations, the number of employed by age groups and gender, the number of vacancies by occupations, the number of employed by levels and fields of education; the number of vacancies by gender, the number of vacancies by occupations, the number of vacancies by occupations, the number of vacancies by occupations, the number of vacancies by educational levels.

185 - 188 formulas reflect the summation equations of the results of the unified labour market sub-models by levels of education.

Formula 185 reflects the calculation of the total number of jobseekers:

$$KDM_{DPJLt} = \left[ \left[ \sum_{k \in Vg} DM_{VgDPJ_{-1}t}^{k} \right]_{1} \cup \left[ \sum_{k \in Vg} DM_{VgDPJ_{-2}t}^{k} \right]_{2} \cup \dots \cup \left[ \sum_{k \in Vg} DM_{VgDPJ_{-}(i-1)t}^{k} \right]_{i-1} \cup \left[ \sum_{k \in Vg} DM_{VgDPJ_{-}it}^{k} \right]_{i} \right], (185)$$

where  $KDM_{DPJL}$  - total number of jobseekers;  $DM_{VgDPJ_i}$  - number of jobseekers at the education level *i*; Vg - age group; D - gender; P - occupation; J - field of education; L - level of education.

Formula 186 reflects the calculation of the total number of employed:

$$KN_{DPJLt} = \left[ \left[ \sum_{k \in Vg} N_{VgDPJ_{-}1t}^{k} \right]_{l} \cup \left[ \sum_{k \in Vg} N_{VgDPJ_{-}2t}^{k} \right]_{2} \cup \dots \cup \left[ \sum_{k \in Vg} N_{VgDPJ_{-}(i-1)t}^{k} \right]_{i-1} \cup \left[ \sum_{k \in Vg} N_{VgDPJ_{-}it}^{k} \right]_{i} \right], \quad (186)$$

where  $KN_{DPJL}$  - total number of employed;  $N_{VgDPJ_i}$  - number of employed at the education level *i*; Vg - age group; D - gender; P - occupation; J - field of education; L - level of education.

Formula 187 reflects the calculation of the total number of vacancies:

$$KVS_{DPJLt} = \left[ \left[ VS_{DPJ_{-1}t} \right]_{1} \cup \left[ VS_{DPJ_{-2}t} \right]_{2} \cup \dots \cup \left[ VS_{DPJ_{-}(i-1)t} \right]_{i-1} \cup \left[ VS_{DPJ_{-}it} \right]_{i} \right], \tag{187}$$

where

KVS<sub>DPJL</sub> - total number of vacancies;
VS<sub>DPJ\_i</sub> - number of vacancies at the education level *i*;
Vg - age group;
D - gender;
P - occupation;
J - field of education;
L - level of education.

Formula 188 reflects the calculation of the number of employed by age groups and gender:

$$N_{VgDt} = \sum_{k \in L} \left( \sum_{i \in P} \sum_{m \in J} N_{VgDPJ_{-it}}^{im} \right)^k, \qquad (188)$$

where

N<sub>VgD</sub> - number of employed by age groups and gender;
N<sub>VgDPJ\_i</sub> - number of employed at the education level *i*;
Vg - age group;
D - gender;
P - occupation;
J - field of education;
L - level of education.

As formulas 185-187 show, in order to obtain the common data, the matrixes of all levels of education are integrated, thereby creating a new matrix. In formulas 185-186 before the integration of matrixes, the number of dimensions is reduced and the age groups are summed up. Formula 187 does not contain such summation as there are no requirements in relation to the age of the employees. The integration of all dimensions into a single matrix is not possible due to technical problems as in this case the number of the matrix elements would exceed one million (1,035 million). In order to reflect the data on employment age, formula 188 provides summing up the employed of different groups of educational levels by age groups and gender. If necessary, the model also allows forming other matrixes on the basis of the above-mentioned dimensions. 185 - 188 formulas provide the calculation of the following key labour market parameters (189 - 196 formulas).

Formula 189 reflects the calculation of the number of jobseekers by levels of education:

$$DM_{Lt} = \sum_{k \in D} \sum_{n \in P} \sum_{m \in J} KDM_{DPJLt}^{knm} , \qquad (189)$$

where  $DM_L$  - number of jobseekers by levels of education;  $KDM_{DPJL}$  - total number of jobseekers; D - gender; P - occupation; J - field of education; L - level of education.

Formula 190 reflects the calculation of the number of jobseekers by gender:

$$DM_{Dt} = \sum_{k \in L} \sum_{n \in P} \sum_{m \in J} KDM_{DPJLt}^{knm}, \qquad (190)$$

where

DM<sub>D</sub> - number of jobseekers by gender; KDM<sub>DPJL</sub> - total number of jobseekers; D - gender; P - occupation; J - field of education; L - level of education.

Formula 191 reflects the calculation of the number of jobseekers by occupations:

$$DM_{Pt} = \sum_{k \in L} \sum_{n \in D} \sum_{m \in J} KDM_{DPJLt}^{knm}, \qquad (191)$$

where  $DM_P$  - number of jobseekers by occupations;  $KDM_{DPJL}$  - total number of jobseekers; D - gender; P - occupation; J - field of education;

L - level of education.

Formula 192 reflects the calculation of the number of employed by occupations:

$$N_{Pt} = \sum_{k \in L} \sum_{i \in D} \sum_{m \in J} K N_{DPJLt}^{kim} , \qquad (192)$$

where

N<sub>P</sub> - number of employed by occupations;
KN<sub>DPJL</sub> - total number of employed;
D - gender;
P - occupation;
J - field of education;
L - level of education.

Formula 193 reflects the calculation of the number of employed by levels and fields of education:

$$N_{LJt} = \sum_{k \in D} \sum_{m \in J} K N_{DPJLt}^{km} , \qquad (193)$$

where

 $N_{\text{LJ}}$  - number of employed by levels and fields of education;

KN<sub>DPJL</sub> - total number of employed;

D - gender;

P - occupation;

J - field of education;

L - level of education.

Formula 194 reflects the calculation of vacancies by gender:

$$VS_{Dt} = \sum_{k \in L} \sum_{i \in P} \sum_{m \in J} KVS_{DPJLt}^{kim} , \qquad (194)$$

where

VS<sub>D</sub> - number of vacancies by gender;

KVS<sub>DPJL</sub> - total number of vacancies;

D - gender;

P - occupation;

J - field of education;

L - level of education.

Formula 195 reflects the calculation of vacancies by occupations:

$$VS_{Pt} = \sum_{k \in L} \sum_{i \in D} \sum_{m \in J} KVS_{DPJLt}^{kim} , \qquad (195)$$

where

 $VS_P$  - number of vacancies by occupations;  $KVS_{DPJL}$  - total number of vacancies; D - gender; P - occupation; J - field of education; L - level of education.

Formula 196 reflects the calculation of vacancies by levels of education:

$$VS_{Lt} = \sum_{k \in P} \sum_{i \in D} \sum_{m \in J} KVS_{DPJLt}^{kim} , \qquad (196)$$

where

VS<sub>L</sub> - number of vacancies by levels of education;

KVS<sub>DPJL</sub> - total number of vacancies;

D - gender;

P - occupation;

J - field of education;

L - level of education.

As the 189 - 196 formulas show, the key labour market parameters have been calculated from the technical elements of the model (185 - 188 formulas), simply by reducing the number of dimensions and summing up the indexes in the selected dimension.

The investigation of the labour market sub-model equation is followed by explanation of the labour market sub-model unified algorithm (Figure 2.47). Calculation equations of the unified algorithm levels are presented in the 197 - 200 formulas.

Formula 197 reflects the calculation equation of the stock of the number of jobseeker:

$$DM_{vgDPJ}(t) = DM_{vgDPJ}(t_{0}) + \int_{t_{0}}^{T} (DMP_{vgDPJ} - PMDM_{vgDPJ} - DMN_{vgDPJ} + MNN_{vgDPJ} + NDM_{vgDPJ} - DMS_{vgDPJ} - DMVS_{(vg-1)DPJ} + DMvS_{vgDPJ} + KOR_{vgDPJ}) dt$$

$$Vg = \begin{cases} \{20_{24}, \dots, 65_{69}, 70_{74}\}, Vg \in NDMvS_{(vg-1)DPJ} \lor DMvS_{vgDPJ} \lor DMN_{vgDPJ} \lor DMS_{vgDPJ} \lor DMS_{vgDP} \lor DMS_{vgD} \lor DMS_{vg} \lor$$

where

DM<sub>VgDPJ</sub> - number of jobseekers;

DMP<sub>VgDPJ</sub> - increase of the number of jobseekers;

PMDM<sub>VgDPJ</sub> - change of occupations in order to get a job;

DMN<sub>VgDPJ</sub> - move from job search to employment;

NDM<sub>VgDPJ</sub> - move from employment to job search;

 $DMS_{VgDPJ}$  - decline in the number of jobseekers;

 $DMvS_{VgDPJ}$  - change of the age structure of jobseekers;

KOR<sub>VgDPJ</sub> - correction;

Vg - age group;

D - gender;

P - occupation;

J - field of education.

Formula 198 reflects the calculation equation of the stock of the number of employed:

$$N_{\rm VgDPJ}(t) = N_{\rm VgDPJ}(t_{0}) + \int_{t_{0}}^{T} (DMN_{\rm VgDPJ} - NPM_{\rm VgDPJ} - NDM_{\rm VgDPJ} + + PMN_{\rm VgDPJ} + NP_{\rm VgDPJ} - NS_{\rm VgDPJ} - NVS_{(Vg-1)DPJ} + NvS_{VgDPJ} - KOR_{VgDPJ} - NKOR_{VgDPJ}) dt , Vg = \begin{cases} \{20_{24}, \dots, 65_{69}, 70_{74}\}, Vg \in NvS_{(Vg-1)DPJ} \lor NvS_{VgDPJ} \\ \{15_{19}, 20_{24}, \dots, 65_{69}, 70_{74}\}, Vg \in DMN_{\rm VgDPJ} \lor NPM_{\rm VgDPJ} \lor , \\ \lor NDM_{\rm VgDPJ} \lor PMN_{\rm VgDPJ} \lor NS_{\rm VgDPJ} \lor KOR_{VgDPJ} \lor NKOR_{VgDPJ} \end{cases}$$
(198)

where

$$\begin{split} N_{VgDPJ} - number of employed; \\ DMN_{VgDPJ} - move from job search to employment; \\ NPM_{VgDPJ} - change of occupation of employed; \\ NDM_{VgDPJ} - move from employment to job search; \\ PMN_{VgDPJ} - employment after the change of occupation; \\ NS_{VgDPJ} - decline in the number of employed due to the change of age structure; \\ NP_{VgDPJ} - decline in the number of employed; \\ NvS_{VgDPJ} - change of the age structure of employed; \\ KOR_{VgDPJ} - correction; \\ NKOR_{VgDPJ} - correction of the structure of employed; \\ Vg - age group; \\ D - gender; \\ P - occupation; \\ J - field of education. \end{split}$$

Formula 199 reflects the calculation equation of the stock of the number of occupied working places:

$$ADV_{\rm DPJ}(t) = ADV_{\rm DPJ}(t_0) + \int_{t_0}^{T} (DVI_{\rm DPJ} - DVT_{\rm DPJ} + ADVK_{\rm DPJ}) dt, \qquad (199)$$

where

ADV<sub>DPJ</sub> - occupied working places;

DVI<sub>DPJ</sub> - the working place is being occupied;

 $DVT_{DPJ}$  - the working place is being released;

ADVK<sub>DPJ</sub> - correction of occupied working places;

D - gender;

P - occupation;

J - field of education.

Formula 200 reflects the calculation equation of the stock of the number of working places:

$$DV_{\rm DPJ}(t) = DV_{\rm DPJ}(t_0) + \int_{t_0}^{T} (DVJ_{\rm DPJ} - DVL_{\rm DPJ}) dt, \qquad (200)$$

 $DV_{DPJ}$  - number of working places;  $DVJ_{DPJ}$  - creation of new working places;  $DVL_{DPJ}$  - decline in the number of working places; D - gender; P - occupation; J - field of education.

As formulas 197 - 200 show, the stock levels of the algorithms are formed by the inflows and outflows. Nature of levels is described below.

The stock of number of jobseekers, i.e., the number of jobseeker is calculated (formula 197) by adding the increase of the number of jobseekers to the initial number of jobseekers (this flow can be positive, i.e., may reflect the direct increase, and can also be negative, indicating the labour reduction), as well as adding unemployed , who move from employment to job search (i.e., employees who have lost their jobs), and subtracting unemployed , who move from job search to employment (have found a job), subtracting the decline in the number of unemployed , who change occupations in order to get a job (have found a job in another occupation), subtracting the decline in number of unemployed due to aging, as well as changing the age structure of the jobseekers and making the correction of the jobseekers in accordance with the synchronization algorithm.

The stock of the number of jobseekers with three flows (move from job search to employment, move from employment to job search and correction) is directly related to the level of the number of employed.

The stock of the number of employed, i.e. the number of employed is calculated (formula 198) by adding the unemployed , who move from job search to employment (have found a job) to the initial number of employed, adding the persons who found a job after the change of occupation, adding the decline in number of employed (this flow can be positive, i.e., may reflect the direct increase, and can also be negative, indicating the labour reduction), and subtracting the number of employed, who changed their occupations (on one hand, this flow subtracts the number of employed, who changed their occupations from the stock of the number of employed, while, on the other hand, adds the number of employed with new occupations, thus, the flow in some groups may be both positive and negative), subtracting the decline in number of unemployed due to aging, as well as changing the age structure of the jobseekers and making the correction of the jobseekers in accordance with the synchronization algorithm.

In total, the stock of the number of jobseekers and employed, and their binding flows model the labour (active population) movement in the labour market: transition from employment to job search and backward, the changes of occupational and age structure, as well as labour growth and reduction.

Figure 2.47 presents two other stocks, namely occupied working places (formula 199) and the number of working places (formula 200). The stock of the number of working places reflects the number of the proposed working places in the national economy, but the stock of the occupied working places reflects how many working places are occupied. Occupied working places result from the initially occupied working places and their affecting changes, i.e., vacation, occupation and correction of occupied working places (appropriate flows). Working places result from the initial working places and affecting changes, i.e., creation and liquidation of working places (appropriate flows).

All flows presented in Figure 2.47 are interrelated. Algorithms presented in Figure 2.47 ensure synchronization of the flows. Employment depends on two factors: labour supply and demand, or, in other words, working places and labour (employed and jobseekers). By

synchronizing working places, occupied working places and the number of employed, the biggest problem is related to the fact that the certain working places can be occupied by partially appropriate staff, that is, for example, the working place can be occupied by an employee with an appropriate occupation, but with nonconforming gender and education. In this case, the structures of working places, occupied working places and employed do not match. The number of vacancies cannot be calculated by subtracting the number of occupied working places from the number of working places. There is a need to include corrections and synchronization algorithms, which are described below along with the flow characterizing equations, in the algorithms scheme (Figure 2.47).

The flows in Figure 2.47 present the results of the algorithms, which combine sub-model algorithms with the most important (central) model parameters. The input data for the flows in Figure 2.47 are mostly related to the sub-model algorithms. The flow constituent equations, showing their relation to the algorithms are presented below (formulas 201, 203 - 216, 219 and 220).

Formula 201 presents the calculation equations of the flow "changes of the age structure of jobseekers":

$$DMvS_{vgDPJt} = NOV_{vgDPJEt} \times DMIs_{vgDPJt},$$
  

$$Vg = \{15\_19, 20\_24, \dots, 65\_69\}, E = active,$$
(201)

where

 $DMvS_{VgDPJ}$  - change of the age structure of jobseekers;  $NOV_{VgDPJE}$  - aging;  $DMIs_{VgDPJ}$  - ratio of jobseekers in active population; E - economic activity group; Vg - age group; D - gender; P - occupation; J - field of education.

As formula 201 shows, changes of the age structure of jobseekers are synchronized with the labour supply module (element "aging"), as well as the ratio of jobseekers in the active population is observed. This formula shows that the active population, from the population labour supply module, is partially aging in the group of jobseekers (the second part is aging in the group of

In order to calculate the changes of the age structure of jobseekers, it is necessary to know the ratio of jobseekers in active population. Its calculation is presented in formula 202:

employed, and this distribution by groups determines the ratio of jobseekers in active population).

$$DMIs_{\text{VgDPJt}} = \begin{cases} \frac{DM_{\text{VgDPJt}} \vee 0}{DM_{\text{VgDPJt}} \vee 0 + N_{\text{VgDPJt}} \vee 0}, (DM_{\text{VgDPJt}} \vee 0 + N_{\text{VgDPJt}} \vee 0) > 0\\ 0, (DM_{\text{VgDPJt}} \vee 0 + N_{\text{VgDPJt}} \vee 0) = 0 \end{cases},$$
(202)

where

DMIs<sub>VgDPJ</sub> - ratio of jobseekers in active population; DM<sub>VgDPJ</sub> - number of jobseekers; N<sub>VgDPJ</sub> - number of employed; Vg - age group; D - gender; P - occupation; J - field of education. The calculation of the group ratio is based on the total calculation principle: the group is divided by the total sum of the group. Formula 202 shows technological specifics of the SDM model. The model analyses various groups, for example, by occupation and age, etc. If the sum of the groups is equal to zero (for example, the certain occupation has no labour, employed and jobseekers), it is not possible automatically to calculate the ratio of the group (it is not possible to divide by zero). In this case, all group elements and the results of the formula are equal to zero. Similarly, formula 202 includes protection against negative numbers among the number of jobseekers and employed people.

The changes of the age structure of employed are related to the changes of the age structure of jobseekers. The calculation of the age structure of the employed is presented in formula 203:

$$NvS_{VgDPJt} = NOV_{VgDPJEt} - DMvS_{VgDPJt},$$
  
$$Vg = \{15\_19,20\_24,...,65\_69\}, E = active,$$
 (203)

where

 $NvS_{VgDPJ}$  - change of the age structure of employed;  $NOV_{VgDPJE}$  - aging;  $DMvS_{VgDPJ}$  - change of the age structure of jobseekers; E - economic activity group; Vg - age group; D - gender; P - occupation; J - field of education.

Formula 203 reflects that the people who have not changed the age group in the group of jobseekers, but have changed in the labour supply module among the active population, change the age group also in the group of employed. Thus, the model provides demographic, labour analysis sub-model synchronization with the labour market sub-model in accordance with the changes of the age structure.

Similarly, the retirement possibilities are ensured, i.e. abandonment of the labour market by reaching the retirement age. Formula 204 presents the calculation of the decline in the number of jobseekers in accordance with the reach of retirement age:

$$DMS_{\rm VgDPJt} = UN_{\rm VgDPJEt} \times DMIs_{\rm VgDPJt}, E = active,$$
(204)

where

 $DMS_{VgDPJ}$  - decline in number of employed;  $UN_{VgDPJE}$  - removal from analysis; E - economic activity group; Vg - age group; D - gender; P - occupation; J - field of education.

As formula 204 shows, the decline in the number of jobseekers is synchronized with the element "removal from analysis", which is in the labour supply module. Labour market abandonment by labour force is calculated in formula 205:

$$NS_{\rm VgDPJt} = UN_{\rm VgDPJt} - DMS_{\rm VgDPJt}, E = active,$$
(205)

 $NS_{VgDPJ}$  - decline in the number of employed due to the change of age structure;  $UN_{VgDPJE}$  - removal from analysis;  $DMS_{VgDPJ}$  - decline in the number of employed; E - economic activity group; Vg - age group; D - gender; P - occupation;

J - field of education.

Formula 205 shows that the people who do not leave the labour market in the group of jobseekers, but leave it in the labour supply module among the active population, also leave it in the group of employed.

Formula 206 presents the flow, which provides the employment of jobseekers according to the current occupation, but formulas 207 and 208 represent the flows that are related to the change of occupation by the population in order to receive a job:

$$DMN_{\text{VgDPJt}} = \sum_{i=1A}^{3B} DMN_{VgDPJt}^{\text{i}} , \qquad (206)$$

where

 $DMN_{VgDPJ}$  - move from job search to employment;  $DMN^{i}_{VgDPJ}$  - occupation of vacancies in the *i* priority; Vg - age group; D - gender; P - occupation; J - field of education.

$$PMDM_{VgDPJt} = \sum_{i=4A}^{5E} PMDM_{VgDPJt}^{i} , \qquad (207)$$

where

 $PMDM_{VgDPJ}$  - change of occupation in order to get a job;  $PMDM_{VgDPJ}^{i}$  - change of occupation in *i* priority; Vg - age group; D - gender;

P - occupation;

J - field of education.

$$PMN_{VgDPJt} = \sum_{i=4A}^{5E} PMN_{VgDPJt}^{i} , \qquad (208)$$

where

 $PMN_{VgDPJ}$  - employment after the change of occupation;  $PMN^{i}_{VgDPJ}$  - occupation of vacancies in *i* priority; Vg - age group; D - gender; P - occupation;

Formulas 206-208 reflect the summing up of the results of employment multi-level algorithm. The algorithm will be investigated later. Levels of the algorithm are numbered with number and letter symbols and formulas show that the first three groups of the algorithm are related to the employment according to the current occupation, but the other two - with the change of occupation of the population in order to receive a job.

Formula 209 reflects the results of the algorithm of the employed occupational mobility operations:

$$NPM_{\rm VgDPJt} = NPMA_{\rm VgDPJt} - NPMV_{\rm VgDPJt}, \qquad (209)$$

where

$$\begin{split} NPM_{VgDPJ} &- change \ of \ occupation \ of \ employed; \\ NPMA_{VgDPJ} &- change \ of \ occupation; \\ NPMV_{VgDPJ} &- occupation \ of \ vacancies \ of \ employed; \\ Vg &- age \ group; \\ D &- gender; \\ P &- occupation; \\ J &- field \ of \ education. \end{split}$$

Formula 209 reflects that in the result of the occupational mobility the same persons, on one hand, leave certain occupations, but on the other hand, fill vacancies in other occupations.

Formula 210 and 211 reflect the flows of the increase of the number of jobseekers and decline in the number of employed. These flows connect jobseekers and employed with labour supply module, labour analysis sub-model; transfer the demographic changes to the labour structure, economic activity, etc. changes to the labour market sub-model.

$$DMP_{\text{VgDPJt}} = UP_{\text{VgDPJEt}} - DMIs_{\text{VgDPJEt}} \times (ISSI_{\text{VgDPJEt}} \vee 0) - (ISSI_{\text{VgDPJEt}} \wedge 0), E = active, \quad (210)$$

where

$$\begin{split} DMP_{VgDPJ} &- \text{ increase of the number of jobseekers;} \\ UP_{VgDPJE} &- \text{ admission for analysis;} \\ DMIs_{VgDPJ} &- \text{ ratio of jobseekers in active population;} \\ ISSI_{VgDPJE} &- \text{ change of population and structure;} \\ Vg &- age group; \\ D &- gender; \\ P &- occupation; \end{split}$$

J - field of education.

$$NP_{\rm VgDPJt} = -(1 - DMIs_{\rm VgDPJt}) \times (ISSI_{\rm VgDPJEt} \vee 0), E = active,$$
(211)

where

 $NP_{VgDPJ}$  - decline in the number of jobseekers;  $DMIs_{VgDPJ}$  - ratio of jobseekers in active population;  $ISSI_{VgDPJE}$  - change of population and structure; E - economic activity group; Vg - age group; D - gender; P - occupation;

The result of formula 210 can be either positive (there is an increase of the number of jobseekers in the appropriate flow) or negative (decline). Formula 211 reflects the decline in employment. This difference is related to the fact that along with the growth of labour, the new labour supplements only the group of jobseekers, but the decline (e.g., mortality) affects both jobseekers and employed. Decline in the number of jobseekers is calculated from the ratio of jobseekers in the active population and positive changes of population and structure in formula 211. The selection of positive changes from the change of population and structure indicates only the decline in population in the analysis group, and is based on the decline in the number of employed. The part which is not observed in the decline process of employed, is observed in the increase of the number of jobseekers (formula 210). In addition, formula 210 shows that the jobseekers in the level of education analysis group are supplemented by persons, who have attained this level of education (admission for analysis), and all negative changes of population and structure. Negative changes of population and structure reflect the increase of the population in the analysis group of the level of education (taking into account the change of emigration or economic activity group). With the growing population in the group, people are included in the circle of jobseekers (if the economic activity status is active), as indicated by formula 210.

According to the algorithm of changes of labour amount, formulas 210 and 211 divide the changes of active population in education, economic activity and demography among jobseekers and employed.

Formula 212 reflects the flow, which makes employed people lose their jobs and move to jobseekers:

$$NDM_{\rm VgDPJt} = ATS1_{VgDPJt} + ATS2_{VgDPJt} , \qquad (212)$$

where

$$\begin{split} NDM_{VgDPJ} &- \text{move from employment to job search;} \\ ATS1_{VgDPJ} &- \text{dismissal in the 1st priority;} \\ ATS2_{VgDPJ} &- \text{dismissal in the 2nd priority;} \\ Vg &- age group; \\ D &- gender; \\ P &- occupation; \\ J &- field of education. \end{split}$$

Formula 212 shows that employed people move from employment to job search in two stages (priorities). The transfer from employment to job search is viewed in detail in the description of the algorithm of decline in the number of vacancies and occupied working places, formula 212 reflects the overall result.

Formula 213 reflects the correction flow of the jobseekers and employed:

$$KOR_{V_{g}DPJt} = DMNKOR_{V_{g}DPJt}, \qquad (213)$$

where  $KOR_{VgDPJ}$  - correction;  $DMNKOR_{VgDPJ}$  - correction of the number of jobseekers and employed; Vg - age group; D - gender; P - occupation; J - field of education.

Formula 213 shows that the correction directly forms (without interference) and follows the synchronization algorithm results.

Formula 214 reflects the correction flow of the structure of employed:

$$NKOR_{V_{g}DPJ_{t}} = NNKOR_{V_{g}DPJ_{t}} - V1NKOR_{V_{g}DPJ_{t}} + NNKOR_{V_{g}DPJ_{t}} - V2NKOR_{V_{g}DPJ_{t}}, \qquad (214)$$

where

NKOR<sub>VgDPJ</sub> - correction of the structure of employed; NNKOR<sub>VgDPJ</sub> - required correction of the structure of employed; V1NKOR<sub>VgDPJ</sub> - age group correction 1; PNNKOR<sub>VgDPJ</sub> - additionally required correction of employed structure; V2NKOR<sub>VgDPJ</sub> - age group correction 2; Vg - age group; D - gender; P - occupation; J - field of education.

Formula 214 processes synchronization algorithm results. Synchronization algorithm defines the correction of the structure of employed so that the number of employed is not negative, which is presented in formula 214. Further formula 212 shows that the age group correction is subtracted from the correction of the employment structure (which is calculated according to the required correction of employed is provided, the age structure is corrected by changing the age group to the previous or next, in order to provide a positive number of employed people. The synchronization algorithm is in detail described in the synchronization algorithm sub-section.

Formula 215 reflects equation of the flow of occupation of working places:

$$DVI_{DPJt} = DVIX_{DPJt} + AVSA_{DPJt} \times \sum_{k \in J} \sum_{i \in D} DVIN_{DPJt}^{ik}, \qquad (215)$$

where

DVI<sub>DPJ</sub> - the working place is being occupied;

DVIX<sub>DPJ</sub> - the working place is being occupied in accordance with vacancies;

AVSA<sub>DPJ</sub> - the structure of remaining vacancies;

 $DVIN_{DPJ}$  - the working place is being occupied in accordance with occupation of vacancies; D - gender;

P - occupation;

J - field of education.

Formula 215, along with formulas 216 and 218 provide analysis of occupied working places. The flow in formula 215 increases the number of occupied working places, the flow in formula 216 reduces the number of the occupied working places, but formula 217 changes (corrects) the employment structure, taking into account the fact that the employment structure does not comply to the vacancy structure (for example, by gender or educational fields). Formula 216 reflects the flow equation of the released working places:

$$DVT_{DPJt} = NS_{DPJt} - \sum_{i \in Vg} NP_{VgDPJt}^{i} + \sum_{i \in Vg} NDM_{VgDPJt}^{i} + \sum_{i \in Vg} NPM_{VgDPJt}^{i}, \qquad (216)$$

where

DVT<sub>DPJ</sub> - the working place is being released;

 $NS_{DPJ}$  - decline in the number of employed due to the change of age structure by occupation, fields of education and gender;

 $NP_{VgDPJ}$  - decline in the number of employed;  $NDM_{VgDPJ}$  - move from employment to job search;  $NPM_{VgDPJ}$  - change of occupation of employed; Vg - age group; D - gender; P - occupation; J - field of education.

Formula 215 operates with the elements of the algorithm of decline in the vacancies and occupied working places, this algorithm is described below. Formula 216 is directly related to the sub-model flows. The only interparameter, which is not covered by the flows or algorithms is the "decline in the number of employed due to the change of age structure by occupation, fields of education and gender" which is derived from "decline in the number of employed due to the change of age structure" by reducing the number of element dimensions (aggregating by age groups) formula 217:

$$NS_{\rm DPJt} = \sum_{i \in Vg} NS_{\rm VgDPJt}^{i} , \qquad (217)$$

where

 $NS_{\text{DPJ}}$  - decline in the number of employed due to the change of age structure by occupations, fields of education and gender;

 $NS_{VgDPJ}$  - decline in the number of employed due to the change of age structure;

Vg - age group;

D - gender;

P - occupation;

J - field of education.

Transferring from the labour analysis to the analysis of the working places, it is necessary to reduce the number of element dimensions in the model, as the labour (supply) requires the analysis by age groups, but the working places (demand) do not have such a requirement. This activity is presented in formula 218. Formula 218 shows the correction calculation equation.

$$ADVK_{\text{DPJt}} = -(ADV_{\text{DPJt}} - DVT_{\text{DPJt}}) \wedge 0 + \begin{cases} \alpha, \alpha + \beta \ge 0\\ -\beta, \alpha + \beta < 0 \end{cases}$$
$$\alpha = \frac{\beta}{\sum_{l \in P} \sum_{k \in J} \sum_{i \in D} \beta^{ikl}} \times \sum_{k \in J} \sum_{i \in D} ((ADV_{\text{DPJt}} - DVT_{\text{DPJt}}) \wedge 0)^{ik}, \qquad \beta = ADV_{\text{DPJt}} + DVI_{\text{DPJt}} - DVT_{\text{DPJt}}, \qquad (218)$$

where

 $ADVK_{DPJ}$  - correction of occupied working places;  $ADV_{DPJ}$  - occupied working places;  $DVT_{DPJ}$  - working place is being released;  $DVI_{DPJ}$  - working place is being occupied.

The calculation of the correction of occupied working places reflects the results of vacancies calculation algorithm, which are analysed in the next subsections.

The number of working places in the system is dependent on development of the national economy: with the development of the national economy, the number of working places increases, but during the crisis, as well as with the growth of the labour productivity, the number of working

places decreases. The equation of the flow of the creation of new working places is presented in formula 219, but the equation of the liquidation flow of the working places is presented in formula 220:

$$DVJ_{\text{DPJt}} = LSP_{\text{DPJt}} = LSP_{\text{DPJ1t}}, L \in i,$$
(219)

where

DVJ<sub>DPJ</sub> - creation of new working places;

LSP<sub>DPJ</sub> - growth of labour demand by occupation and gender at the level of education;

LSP<sub>JLDP</sub> - growth of labour demand by fields of education, levels, occupations and gender; D - gender;

P - occupation;

J - field of education;

L - *i* group of level of education.

$$DVL_{\rm DPJt} = DVLaB_{\rm DPJt} + DVLaD_{\rm DPJt}, \qquad (220)$$

where

DVL<sub>DPJ</sub> - decrease in the number of working places; DVLaB<sub>DPJ</sub> - appropriate decrease in the number of working places; DVLaD<sub>DPJ</sub> - adapted decrease in the number of working places; D - gender; P - occupation; J - field of education.

As formula 219 shows, the creation of new working places at the educational level fully conforms to the increase of labour demand at the educational level. But in order to reflect the elimination of working places, the elimination of working places algorithm has been developed, whose results of performance are reflected in formula 220. Elimination of working places algorithm is analysed in the next subsections.

#### 2.3.1.2. Calculation algorithm of vacancies

The model provides the elements of "occupied working places" and "number of working places" The simplest way of calculation of the number of vacancies would be subtraction of the occupied working places from the total working places. However, it is not practically possible, taking into account that the occupied working places represent the education, gender and occupational structure of the employed, but the working places reflect the demand structure. They may be inconsistent, as some working places may be occupied by people with an inappropriate gender or education. This potential difference defines the need to develop the calculation algorithm of vacancies, which is described in this subsection.

The overall scheme of the calculation algorithm of vacancies is presented in Figure 2.49.



Figure 2.49. The overall scheme of the calculation algorithm of vacancies

The essence of the calculation algorithm of vacancies is the following: the number of appropriate vacancies is calculated from the working places and occupied working places, which is a positive difference of these elements. The number of appropriate vacancies reflects vacancies that are not occupied. At the same time, the negative difference between the working places and occupied working places creates the surplus, which consists of occupied working places that do not have appropriate vacancies (or appropriate working places). In order to calculate the number of vacancies, the surplus of the occupied working places is subtracted from appropriate vacancies, by changing their structure.

The calculation of the surplus of the number of appropriate vacancies and occupied working places is presented in formulas 221 and 222:

$$ATVS_{\text{DPJt}} = (DV_{\text{DPJt}} - ADV_{\text{DPJt}}) \lor 0, \qquad (221)$$

where

 $ATVS_{DPJ}$  - number of appropriate vacancies;  $DV_{DPJ}$  - number of working places;  $ADV_{DPJ}$  - occupied working places; D - gender; P - occupation; J - field of education.

$$NAVS_{Pt} = \sum_{i \in D} \sum_{k \in J} \left( \left( DV_{DPJt} - ADV_{DPJt} \right) \wedge 0 \right)^{ik}, \qquad (222)$$

where

NAVS<sub>P</sub> - surplus of occupied working places; DV<sub>DPJ</sub> - number of working places; ADV<sub>DPJ</sub> - occupied working places; D - gender; P - occupation; J - field of education.

For the calculation of the appropriate number of vacancies and surplus of occupied working places the difference between the working places and occupied working places is used, in the first case it is a positive value, while in the second - negative. In the calculation of the surplus of

occupied working places the obtained surplus has a decreased number of dimensions - surplus reflects only occupations.

The calculation of the appropriate vacancy structure is presented formula 223:

$$ATVSSt_{\text{DPJt}} = \begin{cases} \frac{ATVS_{\text{DPJt}}}{\sum \sum_{i \in D} \sum_{k \in J} ATVS_{\text{DPJt}}^{ik}}, \sum_{i \in D} \sum_{k \in J} ATVS_{\text{DPJt}}^{ik} > 0\\ 0, \sum_{i \in D} \sum_{k \in J} ATVS_{\text{DPJt}}^{ik} \le 0 \end{cases},$$
(223)

where

ATVSSt<sub>DPJ</sub> - structure of appropriate vacancies; ATVS<sub>DPJ</sub> - number of appropriate vacancies; D - gender; P - occupation; J - field of education.

Structure calculation (formula 223) reflects also the protection against situations when it is not possible to calculate the structure (it is not possible to divide by zero); in this case all structural elements are assigned the value of zero.

Calculation of the adapted number of vacancies is presented in formula 224

$$ADAVS_{DPIt} = ATVS_{DPIt} + ATVSSt_{DPIt} \times NAVS_{Pt},$$
 (224)

where

 $ADAVS_{DPJ}$  - adapted number of vacancies;  $ATVS_{DPJ}$  - number of appropriate vacancies;  $ATVSSt_{DPJ}$  - structure of appropriate vacancies;  $NAVS_P$  - surplus of occupied working places; D - gender; P - occupation; J - field of education.

Adapted number of vacancies consists of the appropriate number of vacancies and the surplus of occupied working places, which structure is modified in accordance with the appropriate number of vacancies.

Adapted number of vacancies reflects the number of vacancies. Technically, the number of vacancies is the prescribed value in the model, in the base period. Accordingly, the technical calculation equation of the number of vacancies is the following, formula 225:

$$VS_{\rm DPJt} = \begin{cases} VS_{\rm DPJt0}, t = 0\\ ADAVS_{\rm DPJt}, t > 0 \end{cases}$$
(225)

where

VS<sub>DPJ</sub> - number of vacancies; ADAVS<sub>DPJ</sub> - adapted number of vacancies; D - gender; P - occupation; J - field of education; t - time. Formula 225 reflects that the number of vacancies is not calculated in the base period, but it is the prescribed index.

## 2.3.1.3. Liquidation algorithm of working places

Liquidation of working places is associated with the slowdown of the economy, restructuring of the national economy and labour productivity growth. If there is a working place in the labour market, and it needs to be liquidated, then it is immediately liquidated. The model requires the algorithm of liquidation of working places, as the model does not analyse working places, but working places with occupations, fields of education, gender of potential or actual employees at the level of education. According to the shrinkage of the economy, restructuring of the national economy or productivity growth may indicate the need to liquidate working places by occupations, but the liquidation of working places in accordance with gender and education of the potential or actual employees may be associated with uncertain factors. This uncertainty can be resolved by the algorithm of liquidation of working places, the overall scheme is presented in Figure 2.50.



Figure 2.50. Overall scheme of the algorithm of liquidation of working places

The beginning of the algorithm of liquidation of working places is related to the element "decline in labour demand by occupations and gender at the levels of education". This element is calculated in the labour demand module. Decrease of labour demand is compared with the actual working places, the working places that correspond to the decline in the labour demand, are determined. In the model they are denoted as "appropriate liquidation of working places", i.e. the working places that correspond to the requested decline in the working places are immediately liquidated. Then the difference between the requested decline in the working places and appropriate liquidation of working places is calculated in the model - "surplus". This means working places that are required to be liquidated, but do not exist in the labour market. The formation of this difference or surplus is related to the changes of labour market in accordance with the work requirements of the occupations by gender and education of employees. However, in order to reduce the number of working places according to the requested decline, the number of the remaining working places and their structure are taken into account.

The number of remaining working places is calculated as the difference between the number of working places and appropriate liquidation of the working places. From this index the technical element of "the number of remaining working places by occupations" is calculated, which provides the calculation of the structure of remaining working places. By multiplying the structure of remaining working places (by gender and fields of education) by "surplus" (i.e., the requested for reduction and still non-reduced number of working places) (by occupations), the adapted liquidation of working places is obtained. Adapted liquidation of working places provides decline in working places according to the initially requested decline in working places by occupations, but by adapting the decline, in accordance with the current market structure of the gender and fields of education of the working places.

The calculation of the appropriate liquidation of working places is presented in formula 226:

$$DVLaB_{\rm DPJt} = \begin{cases} LSS_{\rm DPJt}, DV_{\rm DPJt} > LSS_{\rm DPJt} \\ DV_{\rm DPJt}, DV_{\rm DPJt} \le LSS_{\rm DPJt} \end{cases},$$
(226)

where

DVLaB<sub>DPJ</sub> - appropriate decrease of number of working places;

 $LSS_{DPJ}$  - decrease of labour demand by occupation and gender at the level of education;  $DV_{DPI}$  - number of working places;

D - gender;

P - occupation;

J - field of education.

Appropriate liquidation of working places is equal to the low value from the decline in labour demand and number of working places.

Formula 227 reflects the calculation of the element "surplus":

$$ATL_{\rm DPJt} = LSS_{\rm DPJt} - DVLaB_{\rm DPJt}, \qquad (227)$$

where

ATL<sub>DPJ</sub> - surplus;

LSS<sub>DPJ</sub> - decrease of labour demand by occupation and gender at the level of education;  $D_{\text{NJ}}$ 

 $DVLaB_{\text{DPJ}}\xspace$  - appropriate decrease of number of working places;

D - gender;

P - occupation;

J - field of education.

Element "surplus" reflects the number of working places that cannot be immediately automatically liquidated, taking into account the difference between working places and requested decline in the gender and education dimensions.

Formula 228 reflects the calculation of the number of remaining working places:

$$AtDV_{\rm DPJt} = DV_{\rm DPJt} - DVLaB_{\rm DPJt}, \qquad (228)$$

where

AtDV<sub>DPJ</sub> - number of remaining working places;
DV<sub>DPJ</sub> - number of working places;
DVLaB<sub>DPJ</sub> - appropriate decrease of number of working places;
D - gender;
P - occupation;
J - field of education.

The number of the remaining working places reflects the working places remaining after the appropriate liquidation of working places, i.e. how many working places can be liquidated during the operation of algorithm. On its basis the number by occupations (formula 229) and the structure of the remaining working places (formula 230) is calculated:

$$AtDV_{\rm Pt} = \sum_{k \in J} \sum_{i \in D} AtDV_{\rm DPJt}^{ik} , \qquad (229)$$

where

 $AtDV_P$  - number of remaining working places by occupations;  $AtDV_{DPJ}$  - number of remaining working places;

D - gender;

P - occupation;

J - field of education.

$$AtDVS_{DPJt} = \begin{cases} \frac{AtDV_{DPJt}}{AtDV_{Pt}}, AtDV_{Pt} > 0\\ 0, AtDV_{Pt} \le 0 \end{cases},$$
(230)

where

 $AtDVS_{DPJ}$  - structure of remaining working places;  $AtDV_{DPJ}$  - number of remaining working places;  $AtDV_P$  - number of remaining working places by occupations; D - gender; P - occupation; J - field of education.

Formula 230 presents the calculation of the structure of the remaining working places, as well as the model is protected from situations when it is not possible to calculate the structure of the remaining working places (not possible to divide by zero), in this case all structural elements are assigned the value of zero.

The calculation of the adapted liquidation of working places is presented in formula 231:

$$DVLaD_{DPJt} = ATL_{DPJt} \times AtDVS_{DPJt},$$
 (231)

where

DVLaD<sub>DPJ</sub> - adapted decrease of number of working places; ATL<sub>DPJ</sub> - surplus; AtDVS<sub>DPJ</sub> - structure of the remaining working places; D - gender; P - occupation; J - field of education.

Algorithm of liquidation of working places eliminates non-compliance of the decline in the demand and the market structure for working places by gender and level of education, as well as liquidates working places by occupations according to decline in the demand for working places.

## 2.3.1.4. Declining algorithm of vacancies and occupied working places

With the decline in the number of working places the model observes the principle that primarily the number of vacancies is reduced. If the number of vacancies is insufficient to cover the decline in the number of working places, the occupied working places are decreased, the employees move from employment to job search. Algorithm of decline in the number of vacancies and working places divides the decline of working places in the vacancies and occupied working places, as well as determines the release of employees from their working places, taking into account the liquidation of working places. The overall scheme of the algorithm of decline in the number of vacancies and occupied working places is presented in Figure 2.51.



# Figure 2.51. The overall scheme of the algorithm of decline in the number of vacancies and occupied working places

Algorithm of decline in the number of vacancies and occupied working places is based on decline in working places, which was investigated in the previous subsection. In the algorithm the decline in working places is compared with the number of vacancies, vacancies cover the decline in the number of working places as far as possible. Then the decline in the number of vacancies is subtracted from the decline in working places, thus obtaining the necessary decline in occupied working places. According to these indexes, the decline in the number of occupied working places, the release of employed from their work take place, taking into account the decline in the number of working places.

release from work in this algorithm takes place in two phases - in the first phase (the first release priority) the release takes place in accordance with the privileged age structure, i.e., from the market point of view, less required and/or less secured employees (by age structure) are released in the first priority. If the first release priority is not enough sufficient to cover the decline in working places, the proportional (by age structure) release of employed takes place in the second priority.

Release from work in the 1st priority is calculated by multiplying the decline in the number of working places by the privileged age structure value of the release from work; this multiplication is compared to the maximum possible changes of employed. The decline in working places in this operation changes to the decline in the number of employed, i.e., it is extended by the dimension characteristic to employees - age group. The appliance of release privileged age structure does not guarantee that the number of employed will be sufficient by age groups, so it is necessary to compare to the maximum possible changes of employed. After the release from work in the 1st priority, the decline in the number of occupied working places is calculated in the second priority, i.e. the difference between the decline in the number of occupied working places and release from work in the 1st priority. Simultaneously with the release from work in the 1st priority and the maximum possible changes of employed, the number of employed in the 2nd priority of release is calculated; this is the difference between the constituent elements. The number of employed in the 2nd priority. By multiplying the age structure of employed for the 2nd priority. By multiplying the age structure of employed in the 2nd priority by the decline in the working places in the 2nd priority, the release from work in the 2nd priority is calculated.

Release from work in accordance with the decline in the number of working places is the summary result of two release levels.

The calculation of the decline in the number of vacancies is presented in formula 232:

$$VSS_{DPJt} = DVL_{DPJt} \land VS_{DPJt}, \qquad (232)$$

where

 $VSS_{DPJ}$  - decline in the number of vacancies;  $DVL_{DPJ}$  - liquidation of working places;  $VS_{DPJ}$  - number of vacancies; D - gender; P - occupation; J - field of education.

Decline in the number of working places provides e less value of the number of vacancies and liquidation of working places.

Formula 233 reflects the calculation of the decrease of occupied working places.

$$ADVS_{DPJt} = DVL_{DPJt} - VSS_{DPJt}, \qquad (233)$$

where

ADVS<sub>DPJ</sub> - decline in the number of remaining working places;

DVL<sub>DPJ</sub> - liquidation of working places;

VSS<sub>DPJ</sub> - decline in the number of vacancies;

D - gender;

P - occupation;

J - field of education.

Formula 234 reflects the calculation of the release from work in the 1 priority:

$$ATS1_{V_gDPJ_t} = \left(ADVS_{DPJ_t} \times ATSS_{V_g}\right) \wedge NMI_{V_gDPJ_t},$$
(234)

where

 $ATS1_{VgDPJ}$  - release from work in the 1st priority;  $ADVS_{DPJ}$  - decline in the number of remaining working places;  $ATSS_{Vg}$  - privileged age structure of the release from work;  $NMI_{VgDPJ}$  - maximum possible changes of employed; Vg - age group; D - gender;

P - occupation;

Release from work in the 1st priority is calculated by multiplying the decline in the number of working places by the age structure of the release from work, this multiplication is compared with the maximum possible changes in employment, and the minimum index is selected.

Formula 235 presents the calculation of decline in number of occupied working places in the 2nd priority:

$$ADVS2_{DPJt} = ADVS_{DPJt} - \sum_{i \in Vg} ATS1^{i}_{VgDPJt} , \qquad (235)$$

where

ADVS2<sub>DPJ</sub> - decline in the number of occupied working places in the 2nd priority; ADVS<sub>DPJ</sub> - decline in the number of remaining working places; ATS1<sub>VgDPJ</sub> - release from work in the 1st priority; Vg - age group; D - gender; P - occupation; J - field of education.

The number of occupied working places in the 2nd priority is calculated as the difference between the decline in the number of occupied working places and release from work in the 1st priority. Release from work in the 1st priority has the reduced number dimensions, taking into account that the working places, in contrast to employed persons, are not analysed by age groups.

Formula 236 reflects the calculation of the number of employed in the 2nd priority of release from work:

$$NMI2_{V_gDPJ_t} = NMI_{V_gDPJ_t} - ATS1_{V_gDPJ_t}, \qquad (236)$$

where

$$\begin{split} NMI2_{VgDPJ} \text{ - number of employed in the 2nd priority of release from work;} \\ NMI_{VgDPJ} \text{ - maximum possible changes of employed;} \\ ATS1_{VgDPJ} \text{ - release from work in the 1st priority;} \\ Vg \text{ - age group;} \\ D \text{ - gender;} \\ P \text{ - occupation;} \\ J \text{ - field of education.} \end{split}$$

Release of employed in the 2nd priority is based on the calculation of the age structure of employed in the 2nd priority, formula 237:

$$NvS2_{vgt} = \begin{cases} \sum_{k \in Dn \in P} \sum_{m \in J} NMI2_{vgDPJt}^{knm} \\ \sum_{i \in Vg} \sum_{k \in D} \sum_{n \in P} \sum_{m \in J} NMI2_{vgDPJt}^{iknm} \\ 0, \sum_{i \in Vg} \sum_{k \in Dn \in P} \sum_{m \in J} NMI2_{vgDPJt}^{iknm} \\ 0, \sum_{i \in Vg} \sum_{k \in Dn \in P} \sum_{m \in J} NMI2_{vgDPJt}^{iknm} \\ \leq 0 \end{cases},$$
(237)

where

 $NvS2_{Vg}$  - age structure of employed in the 2nd priority;  $NMI2_{VgDPJ}$  - number of employed in the 2nd priority of release from work; Vg - age group; D - gender; P - occupation; J - field of education.

Formula 238 reflects the release from work in the 2nd priority:

$$ATS2_{VgDPJt} = ADVS2_{DPJt} \times NvS2_{Vgt}, \qquad (238)$$

where

 $\begin{array}{l} ATS2_{VgDPJ} \mbox{-} release from work in the 2nd priority;} \\ ADVS2_{DPJ} \mbox{-} decrease in the number of occupied working places for the 2nd priority;} \\ NvS2_{Vg} \mbox{-} age structure of employed in the 2nd priority;} \\ Vg \mbox{-} age group; \\ D \mbox{-} gender; \\ P \mbox{-} occupation; \\ J \mbox{-} field of education. \end{array}$ 

Release from work in the 2nd priority along with the release from work in the 1st priority form the total release from work, which is investigated in formula 212.

Algorithm of decline in the number of vacancies and occupied working places has been examined, but the elements, which link it to the rest of the model algorithms, are not reflected - among them the privileged age structure of the release from work (formula 239) and the maximum possible changes of employed (formula 240):

$$ATSS_{Vg} = \frac{\frac{1}{PSt_{Vg}}}{\sum_{i \in Vg} \frac{1}{PSt_{Vg}^{i}}},$$
(239)

where

 $ATSS_{Vg}$  - privileged age structure of the release from work; PSt<sub>Vg</sub> - preferential age structure; Vg - age group;

The element "privileged age structure of the release from work" reflects the view of employers on the privileged age structure of employees, releasing the employees from work. This Element is associated with an element (and its opposite at the same time) "privileged age structure", which determines the privileges of employees by getting a job. Both these elements are not dependent on time, and despite the fact that one element in the model is expressed through the other, they are constant values. Their values are presented in annex A5.

The indexes of privileged age structure determine that, primarily, the employees of average age get a job, and employees above and below the average age are released from work.

Formula 240 presents the calculation of the element "maximum possible changes of employed":

$$NMI_{vgDPJt} = NMI2_{vgDPJt} \vee \begin{cases} N_{vgDPJt} + DMN_{vgDPJt} + PMN_{vgDPJt} - NPM_{vgDPJt} - NS_{vgDPJt} + NP_{vgDPJt} - DMvSv_{vgDPJt}, \\ Vg \in \{vg15\_19, ..., vg65\_69, \\ N_{vgDPJt} + DMN_{vgDPJt} + PMN_{vgDPJt} - NPM_{vgDPJt} - NS_{vgDPJt} + NP_{vgDPJt}, Vg = vg70\_74 \\ Vg \in \{vg15\_19, ..., vg70\_74\}, \end{cases}$$
(240)

$$\begin{split} NMI_{VgDPJ} &- maximum possible changes of employed; \\ NMI2_{VgDPJ} &- maximum possible changes of employed from LS; \\ N_{VgDPJ} &- number of employed at the level of education; \\ DMN_{VgDPJ} &- move from job search to employment; \\ PMN_{VgDPJ} &- employment after the change of occupation; \\ NPM_{VgDPJ} &- change of occupation of employed; \\ NS_{VgDPJ} &- decrease in the number of jobseekers due to the change of age structure; \\ NP_{VgDPJ} &- decline in the number of jobseekers; \\ NvS_{VgDPJ} &- changes of the age structure of employed; \\ Vg &- age group; \\ D &- gender; \\ P &- occupation; \\ J &- field of education. \end{split}$$

The task of the element of maximum possible changes of employed is to provide the sequential changes of the stocks variables.

As formula 240 shows, the maximum possible changes of employed are related to the appropriate stock variable "number of employed at the level of education" and the related flows, including the decline in the number of employed, transfer from job search to employment, change of occupations and age structures. The maximum possible changes of employed are calculated in two age groups: up to the age of 69 years and after the age of 70 years. This division has a technical nature of the model. The model for the group after the age of 70 years does not analyse the changes of the age structure as the largest number of population, having reached this age groups; the other elements of the equation are equal.

Additionally, formula 240 represents that the maximum possible changes of employed are limited (cannot be larger, exceed) by the element of "maximum possible changes of employed from LS". This element does not represent the maximum possible changes from the stock and flows of the employed, but from the perspective of the labour supply. It is understood that the changes of employed people cannot exceed labour changes. The calculation of the element of "maximum possible changes of employed from LS" is presented in formula 241:

$$NMI2_{\rm VgDPJt} = (1 - DMIs_{\rm VgDPJt}) \times IZM_{\rm VgDPJEt}, E = active, \qquad (241)$$

where

 $NMI2_{VgDPJ}$  - maximum possible changes of employed from LS;  $DMIs_{VgDPJ}$  - ratio of jobseekers in active population;  $IZM_{VgDPJE}$  - maximum of changes; E - economic activity group; Vg - age group; D - gender; P - occupation;

Maximum possible changes of employed from LS reflect the maximum potential changes of the active labour that are related to employed population (the remaining part is related to the jobseekers).

The algorithm of decline in the number of vacancies and occupied working places regulates the labour market balance with the decline in the labour demand.

#### 2.3.1.5. **Employment and occupational mobility algorithm**

Algorithm of employment and occupational mobility is complex, it consists of several subalgorithms. The overall scheme of the algorithm of employment and occupational mobility is presented in Figure 2.52.

Occupational mobility of employed. Employed people by changing their occupations occupy vacancies; the new vacancies are being organized at that moment.



Are accepted for employment

## Figure 2.52. The overall scheme of the algorithm of employment and occupational mobility

As Figure 2.52 shows, employment (including recruitment) is carried out in several stages. First of all, the jobseekers, whose gender, occupation and education correspond to vacancies, are employed in the first priority. It is important to note that each stage, reflected in Figure 2.52, is divided into two sub-stages, where the employment by age is analysed: initially, in the first substage is the employment according to privileged age structure (see Annex A5), and afterwards, in the second sub-stage - according to the age structure of the market supply. Model explanation substages are designated by letters A and B. In total with the priorities presented in Figure 2.52, they create the 8 sub-stage (from 1A to 4B, the further stages are used only in the absence of staff, accordingly at the fifth and sixth stages the employers do not have the opportunity to choose employees by age).

If the first priority does not cover all vacancies, then those jobseekers are accepted for employment in the second priority, whose occupations and fields of education correspond to their vacancies, in this case the employers will not have any possibility to choose employees by gender.

In the third priority those jobseekers are accepted for employment, whose occupations (and appropriate experience) correspond to their vacancies, in this case gender and field of education are not important.

In the fourth priority those jobseekers are accepted for employment, whose fields of education correspond to the field of education of the vacancy, and, therefore, the attained education corresponds to the vacancy of occupation. In this case, considering that it is possible to work by occupation with the certain education, the previous occupation of the employees is not taken into account. Similarly, the gender is not taken into account as well.

In the fifth priority the jobseekers occupy vacancies in similar occupations, i.e., there is a change of occupation in order to get a job. The field of education, gender and age of employees are not taken into account.

The sixth priority determines the occupational mobility of employees. If certain occupations have better work conditions, there are vacancies and lack of employees, as well as it is also possible to change the occupation, the employees can change their occupations, occupying better occupations and releasing working places in unattractive occupations.

Employment and algorithm of occupational mobility analyses not only the employment procedure, but also regulates the ability of new employees to keep their job and determines the decline in the number of vacancies after employment.

In the employment sub-models (stages) the unified scheme is used, which is reflected in Figure 2.53.



Figure 2.53. The unified scheme of the employment sub-algorithms (stages)

The unified algorithm starts with the number of jobseekers and vacancies in the given priority. By distributing the number of vacancies according to the privileged age structure, vacancies are compared with the number of jobseekers, and the lowest index is selected. The resulting figure is adjusted according to wage correlation index of the supply and demand and geographical compliance index of the supply and demand, resulting in vacancy occupation rate (certain priority). It is important to note that the geographical correlation indexes are constant and equal to 1 in the model, i.e., the supplied geography in the labour market corresponds to the demand geography.

From the total number of jobseekers and occupation of vacancies in the priority the number of jobseekers for the next priority is calculated (i.e. the number of jobseeker who have not found jobs in the current priority). By analogy, from the number of vacancies in the priority and occupation of vacancies in the priority the number of vacancies for your next priority is calculated.

The privileged age structure is not used for the employment in the next priority; instead the age structure of jobseekers is calculated. Next, by analogy to the first stage, on the basis of the number of vacancies, structure and number of jobseekers, the occupation of vacancies is calculated.

Two-stage scheme is repeated n times.

Figure 2.53 reflects the unified algorithm scheme, but each priority has a slight deviation from it. Deviations are related to the need to establish a variety of jobseeker structures, i.e., by gender, fields of education, and occupations. The scheme presented in figure 2.53 fully reflects the first stage of employment; the equations are presented in formulas 242 -246.

$$DMN_{VgDPJt}^{1A} = PPAAK^{1A} \times PPGAK^{1A} \times \begin{cases} DM_{VgDPJt}, VS_{DPJt} \times PS_{Vg} \ge DM_{VgDPJt} \\ VS_{DPJt} \times PS_{Vg}, VS_{DPJt} \times PS_{Vg} < DM_{VgDPJt} \end{cases},$$
(242)

where

 $DMN^{1A}_{VgDPJ}$  - occupation of vacancies in the 1A priority;  $PPAAK^{1A}$  - wages compliance coefficient of demand and supply in the 1A priority;  $PPGAK^{1A}$  - geographical compliance coefficient of demand and supply in the 1A priority;  $DM_{VgDPJ}$  - number of jobseekers at the level of education;  $VS_{DPJ}$  - number of vacancies at the level of education;  $PS_{Vg}$  - preferential age structure; Vg - age group; D - gender;

P - occupation;

J - field of education.

$$VS_{\text{DPJt}}^{1\text{A}} = VS_{\text{DPJt}} - \sum_{i \in Vg} DMN_{Vg\text{DPJt}}^{1\text{A}}, \qquad (243)$$

where

 $VS_{DPJ}^{1B}$  - number of vacancies in the 1B priority;  $VS_{DPJ}$  - number of vacancies at the level of education;  $DMN_{VgDPJ}^{1A}$  - occupation of vacancies in the 1A priority; Vg - age group; D - gender;

P - occupation;

J - field of education.

$$DM_{VgDPJt}^{1B} = DM_{VgDPJt} - DMN_{VgDPJt}^{1A}, \qquad (244)$$

where

 $DM^{1B}_{VgDPJ}$  - number of jobseekers in the 1B priority;  $DM_{VgDPJ}$  - number of jobseekers at the level of education;  $DMN^{1A}_{VgDPJ}$  - occupation of vacancies in the 1A priority; Vg - age group; D - gender;

P - occupation;

$$DMvS_{Vgt}^{IB} = \begin{cases} \sum_{k \in Dn \in P} \sum_{m \in J} DM_{VgDPJt}^{IB} {}^{knm}, \sum_{i \in Vg} \sum_{k \in Dn \in P} \sum_{m \in J} DM_{VgDPJt}^{IB} {}^{iknm}, \sum_{i \in Vg} \sum_{k \in Dn \in P} \sum_{m \in J} DM_{VgDPJt}^{IB} {}^{iknm} > 0\\ 0, \sum_{i \in Vg} \sum_{k \in Dn \in P} \sum_{m \in J} DM_{VgDPJt}^{IB} {}^{iknm} \le 0 \end{cases},$$
(245)

 $DMvS^{1B}_{Vg}$  - age structure of employed in the 1B priority;  $DM^{1B}_{VgDPJ}$  - number of jobseekers in the 1B priority; Vg - age group; D - gender; P - occupation; J - field of education.

$$DMN_{V_{g}DPJ_{t}}^{1B} = PPAAK^{1B} \times PPGAK^{1B} \times \begin{cases} DM_{V_{g}DPJ_{t}}^{1B}, VS_{DPJ_{t}}^{1B} \times DM_{V}S_{V_{gt}}^{1B} \ge DM_{V_{g}DPJ_{t}}^{1B} \\ VS_{DPJ_{t}}^{1B} \times DM_{V}S_{V_{gt}}^{1B}, VS_{DPJ_{t}} \times DM_{V}S_{V_{gt}}^{1B} < DM_{V_{g}DPJ_{t}}^{1B} \end{cases}, \quad (246)$$

where

 $DMN^{1B}_{VgDPJ}$  - occupation of vacancies in the 1B priority;  $PPAAK^{1B}$  - wages compliance coefficient of demand and supply in the 1B priority;  $PPGAK^{1B}$  - geographical compliance coefficient of demand and supply in the 1B priority;  $DM^{1B}_{VgDPJ}$  - number of jobseekers in the 1B priority;  $VS^{1B}_{DPJ}$  - number of vacancies in the 1B priority;  $DMvS^{1B}_{Vg}$  - age structure of jobseekers in the 1B priority; Vg - age group; D - gender; P - occupation; J - field of education.

The equations of the first stage employment (formulas 242 - 246) are simple and do not require further explanation. By using the number of jobseekers and vacancies, they determine the correspondence of the number of jobseekers and vacancies, appropriate employment, as well as the remaining number of jobseekers and vacancies for the subsequent stages.

Figure 2.54 presents the second priority sub-model of employment, where the market requirements for employees (of certain gender) are not provided, but are implemented in accordance with the supply.



Figure 2.54. Sub-model of the employment of the second priority

Comparing the second priority sub-model of employment (Figure 2.54) with the unified scheme (Figure 2.53), the following differences are evident: from jobseekers the gender structure is calculated; vacancies are analysed without the gender requirements; in the occupation of vacancies the number of jobseekers, the number of vacancies, gender structure of jobseekers are considered and, by analogy with the unified scheme - age structure, wage and geographical compliance indexes.

In this scheme vacancies may be occupied by inappropriate employees, that is, instead of men women can be employed. After occupation of vacancies it is necessary to calculate the number of vacancies for the next priority, i.e. the number of vacancies. Considering the fact that vacancies may be occupied by inappropriate employees, it is not possible to obtain the number of vacancies by subtracting the number of occupied vacancies from the initial number of vacancies in the priority. In order to determine the number of vacancies adequately for the next priority, the submodels determine inappropriately occupied vacancies, which are adjusted according to the initial demand, which allows determining the number of vacancies adequately.

To explore inappropriately occupied vacancies, it is determined, which vacancies should be occupied in accordance with the initial gender structure of vacancies (or gender demand) at the actual vacancy occupation level. comparing the occupation of vacancies by gender demand with the initial number of vacancies, the nonconformity is determined. Nonconformity is divided by gender - women and men. Then the gender of employees is changed for inappropriately occupied vacancies in the model, i.e. if the vacancy has been designed for men, but occupied by a woman, it is considered that it is still occupied by a man (i.e., a woman has been replaced by a man) (the gender remains constant only for the registration of employed and jobseekers). This procedure allows reducing the number of vacancies of the same gender, if they are occupied by jobseekers of the other gender; at the same time the number of vacancies of the other gender remains unchanged.

Equations of employment in the second stage are presented in formulas 247 - 259.

$$DM_{V_{g}DPJ_{t}}^{2A} = DM_{V_{g}DPJ_{t}}^{1B} - DMN_{V_{g}DPJ_{t}}^{1B}, \qquad (247)$$

- $DM^{2A}_{VgDPJ}$  number of jobseekers in the 2A priority;  $DM^{1B}_{VgDPJ}$  number of jobseekers in the 1B priority;  $DMN^{1B}_{VgDPJ}$  occupation of vacancies in the 1B priority; Vg - age group; D - gender;
- P occupation;
- J field of education.

$$DMdS_{Dt}^{2A} = \begin{cases} \frac{\sum_{k \in Vgn \in P} \sum_{m \in J} DM_{VgDPJt}^{2A} {}^{knm}}{\sum_{i \in D} \sum_{k \in Vgn \in P} \sum_{m \in J} DM_{VgDPJt}^{2A} {}^{iknm}}, \sum_{i \in D} \sum_{k \in Vgn \in P} \sum_{m \in J} DM_{VgDPJt}^{2A} {}^{iknm} > 0\\ 0, \sum_{i \in D} \sum_{k \in Vgn \in P} \sum_{m \in J} DM_{VgDPJt}^{2A} {}^{iknm} \le 0 \end{cases},$$
(248)

where

 $DMdS^{2A}_{D}$  - gender structure of jobseekers in the 2A priority;  $DM^{2A}_{VgDPJ}$  - number of jobseekers in the 2A priority; Vg - age group; D - gender; P - occupation; J - field of education.

$$VS_{\text{DPJt}}^{2A} = VS_{\text{DPJt}}^{1B} - \sum_{i \in Vg} DMN_{Vg\text{DPJt}}^{1B}{}^{i}, \qquad (249)$$

where

 $VS^{2A}_{DPJ}$  - number of vacancies in the 2A priority;  $VS^{1B}_{DPJ}$  - number of vacancies in the 1B priority;  $DMN^{1B}_{VgDPJ}$  - occupation of vacancies in the 1B priority; Vg - age group; D - gender; P - occupation; J - field of education.

$$VSbD_{\rm PJt}^{2\rm A} = \sum_{i \in D} VS_{\rm DPJt}^{2\rm A}^{i} , \qquad (250)$$

where

VSbD<sup>2A</sup><sub>PJ</sub> - number of vacancies in the 2A priority without gender requirements; VS<sup>2A</sup><sub>DPJ</sub> - number of vacancies in the 2A priority; D - gender;

P - occupation;

$$DMN_{VgDPJt}^{2A} = PPAAK^{2A} \times PPGAK^{2A} \times \times \begin{cases} DM_{VgDPJt}^{2A}, VSbD_{PJt}^{2A} \times PS_{Vgt} \times DMdS_{Dt}^{2A} \ge DM_{VgDPJt}^{2A}, \\ VSbD_{PJt}^{2A} \times DMvS_{Vet}^{2A} \times DMdS_{Dt}^{2A}, VSbD_{PJt} \times PS_{Vet} \times DMdS_{Dt}^{2A} < DM_{VeDPJt}^{2A} \end{cases}$$
(251)

 $\begin{array}{l} {\rm DMN}^{2A}{}_{VgDPJ} \mbox{-} occupation of vacancies in the 2A priority;} \\ {\rm PPAAK}^{2A} \mbox{-} wages compliance coefficient of demand and supply in the 2A priority;} \\ {\rm PPGAK}^{2A} \mbox{-} geographical compliance coefficient of demand and supply in the 2A priority;} \\ {\rm DM}^{2A}{}_{VgDPJ} \mbox{-} number of jobseekers in the 2A priority;} \\ {\rm VSbD}^{2A}{}_{PJ} \mbox{-} number of vacancies in the 2A priority without gender requirements;} \\ {\rm PS}_{Vg} \mbox{-} preferential age structure;} \\ {\rm DMdS}^{2A}{}_{D} \mbox{-} gender structure of jobseekers in the 2A priority;} \\ {\rm Vg \-} age group; \\ {\rm D \-} gender; \\ {\rm P \-} occupation; \\ {\rm J \-} field of education. \end{array}$ 

As formulas 247 - 251 show, the second priority sub-model of employment is essentially repeating the first priority sub-model of employment. Existing differences are related to the observance of the age structure of jobseekers. Then the gender nonconformity balancing equations are presented (formulas 252 - 259).

$$VSdS_{Dt}^{2A} = \begin{cases} \frac{\sum_{n \in P} \sum_{m \in J} DM_{DPJt}^{2A^{nm}}}{\sum_{i \in D} \sum_{n \in P} \sum_{m \in J} DM_{DPJt}^{2A^{inm}}}, \sum_{i \in D} \sum_{n \in P} \sum_{m \in J} DM_{DPJt}^{2A^{inm}} > 0\\ 0, \sum_{i \in D} \sum_{n \in P} \sum_{m \in J} DM_{DPJt}^{2A^{inm}} \le 0 \end{cases},$$
(252)

where

VSdS<sup>2A</sup><sub>D</sub> - gender structure of vacancies in the 2A priority; VS<sup>2A</sup><sub>DPJ</sub> - number of vacancies in the 2A priority; D - gender; P - occupation; J - field of education.

$$DMNbD_{PJt}^{2A} = \sum_{m \in Di \in Vg} DMN_{VgDPJt}^{2A}^{im}, \qquad (253)$$

where

 $DMNbD^{2A}_{PJ}$  - occupation of vacancies in the 2A priority without the gender requirements;  $DMN^{2A}_{VgDPJ}$  - occupation of vacancies in the 2A priority; Vg - age group; D - gender; P - occupation;

$$DMNpD_{DPJt}^{2A} = VSdS_{Dt}^{2A} \times DMNbD_{PJt}^{2A} , \qquad (254)$$

 $DMNpD^{2A}_{DPJ}$  - occupation of vacancies in the 2A priority by gender demand;  $VSdS^{2A}_{D}$  - gender structure of vacancies in the 2A priority;  $DMNbD^{2A}_{PJ}$  - occupation of vacancies in the 2A priority without the gender requirements; D - gender; P - occupation;

J - field of education.

$$DPN_{DPJt}^{2A} = \left(VS_{DPJt}^{2A} - DMNpD_{DPJt}^{2A}\right) \wedge 0, \qquad (255)$$

where

DPN<sup>2A</sup><sub>DPJ</sub> - noncompliance of gender of occupation of vacancies in the 2A priority; VS<sup>2A</sup><sub>DPJ</sub> - number of vacancies in the 2A priority; DMNpD<sup>2A</sup><sub>DPJ</sub> - occupation of vacancies in the 2A priority by gender demand; D - gender; P - occupation; J - field of education.

The element "Noncompliance of gender of occupation of vacancies of the 2A" reflects the difference between the initial and calculated number of vacancies. This element is negative, i.e., less than zero. In the following two equations (formulas 256 -257) it is divided by gender.

$$DPNv_{\rm PJt}^{2\rm A} = DPN_{\rm DPJt}^{2\rm A}, \, \mathrm{D} = \mathrm{men},$$
(256)

where

 $DPNv^{2A}_{PJ}$  - noncompliance of gender of occupation of vacancies in the 2A priority for men;  $DPN^{2A}_{DPJ}$  - noncompliance of gender of occupation of vacancies in the 2A priority; D - gender;

P - occupation;

J - field of education.

$$DPNs_{PJt}^{2A} = DPN_{DPJt}^{2A}, D =$$
women, (257)

where

 $DPNs^{2A}_{PJ}$  - noncompliance of gender of occupation of vacancies in 2A priority for women;  $DPN^{2A}_{DPJ}$  - noncompliance of gender of occupation of vacancies in 2A priority;

D - gender;

P - occupation;

J - field of education.

$$DMNI_{DPJt}^{2A} = \begin{bmatrix} -DPNv_{PJt}^{2A}, D = \text{women} \\ U \\ -DPNs_{PJt}^{2A}, D = \text{men} \end{bmatrix},$$
(258)

where

DMNI<sup>2A</sup><sub>DPJ</sub> - changes of gender demand of occupation of vacancies in the 2A priority; DPNv<sup>2A</sup><sub>PJ</sub> - noncompliance of gender of occupation of vacancies in the 2A priority for men;  $DPNs^{2A}_{PJ}$  - noncompliance of gender of occupation of vacancies in the 2A priority for women;

D - gender; P - occupation; J - field of education.

Formula 258 reflects the formation of new matrixes by changing the gender for the inappropriately occupied vacancies. It is important to note that the element values of 255 - 257 formulas are negative, i.e., less than zero. This indicates the lack of vacancies of certain gender. In formula 258 the minus sign has been changed to plus sign, as the inappropriate gender has been changed to the appropriate, i.e., the lack is liquidated.

$$VS_{\rm DPJt}^{2\rm B} = \left( VS_{\rm DPJt}^{2\rm A} - DMNpD_{\rm DPJt}^{2\rm A} \right) \lor 0 - DMNI_{\rm DPJt}^{2\rm A},$$
(259)

where

 $VS_{DPJ}^{2B}$  - number of vacancies in the 2B priority;  $VS_{DPJ}^{2A}$  - number of vacancies in the 2A priority;  $DMNpD_{DPJ}^{2A}$  - occupation of vacancies in the 2A priority by gender demand;  $DMNI_{DPJ}^{2A}$  - changes of gender demand of occupation of vacancies in the 2A priority; D - gender; P - occupation; J - field of education.

Number of vacancies for the next 2B priority is calculated by subtracting the changes of the gender vacancies from the positive (larger than zero value) initial and calculated difference of the number of vacancies. Formula 259 reflects the positive part of the difference between initial and calculated number of vacancies. The negative part of this difference is selected in formula 255, and the gender changes of vacancies are calculated. Thus, formula 259 reflects the remaining vacancies of the second stage after employment, taking into account the changes of gender structure.

2B priority sub-model equations repeat the 2A priority equations (247 - 259 formulas), except for the fact that instead of the privileged age structure in the 2B priority the age structure of jobseekers is used (the calculation is represented by formula 245). Considering the equation duplication, they are not repeatedly reflected here.

Figure 2.55 reflects the third priority sub-model of the sub-model, where the market demands by gender and education of employees are not provided, but implemented in accordance with the supply.



Figure 2.55. Sub-model of the employment of the third priority

The Figure 2.55 shows that the third priority sub-model of employment up to vacancy occupation element actually repeats the second priority sub-model. The differences are related to the observance of the additional structure (education structure).

After the occupation of vacancies the procedure of decline in the number of free vacancies is completely different. Vacancies are occupied by employees by certain gender and fields of education, occupation of jobseekers correspond to the demanded occupations in vacancies. Gender and fields of education in vacancies. In order to balance jobseekers and vacancies, the occupied vacancies are registered without gender and education; further the gender and education structures are determined for the decline in the number of vacancies. Gender and education structures of vacancies are calculated twice: the first time - in accordance with the initial number of vacancies (i.e., according to the initial demand), and the second time - to distribute the noncompliant occupation structures of vacancies (i.e., by remaining or available on the market vacancies). If the gender and education structures of vacancies, it is provided by the structure of vacancies, which observes not only the gender and education, but also the occupation. More detailed explanation of this procedure is given, by reflecting the procedure equations.

The equations of the third phase employment are presented in formulas 260 - 277.

$$DM_{V_{g}DPJ_{t}}^{3A} = DM_{V_{g}DPJ_{t}}^{2B} - DMN_{V_{g}DPJ_{t}}^{2B}, \qquad (260)$$

 $DM^{3A}_{VgDPJ}$  - number of jobseekers in the 3A priority;  $DM^{2B}_{VgDPJ}$  - number of jobseekers in the 2B priority;  $DMN^{2B}_{VgDPJ}$  - occupation of vacancies in the 2B priority; Vg - age group; D - gender; P - occupation;

J - field of education.

$$DMdS_{Dt}^{3A} = \frac{\sum_{k \in Vg} \sum_{n \in P} \sum_{m \in J} DM_{VgDPJt}^{3A} {}^{knm}}{1 \vee \sum_{i \in D} \sum_{k \in Vg} \sum_{n \in P} \sum_{m \in J} DM_{VgDPJt}^{3A} {}^{iknm}},$$
(261)

where

 $DMdS^{3A}_{D}$  - gender structure of jobseekers in the 3A priority;  $DM^{3A}_{VgDPJ}$  - number of jobseekers in the 3A priority; Vg - age group; D - gender;

P - occupation:

J - field of education.

$$DMizS_{Jt}^{3A} = \frac{\sum_{k \in Vg} \sum_{n \in P} \sum_{m \in D} DM_{VgDPJt}^{3A}^{Nnm}}{1 \vee \sum_{i \in D} \sum_{k \in Vg} \sum_{n \in P} \sum_{m \in J} DM_{VgDPJt}^{3A}^{Nnm}},$$
(262)

where  $DMizS^{^{3A}}J$  - education structure of jobseekers in the 3A priority;  $DM^{^{3A}}V_{gDPJ}$  - number of jobseekers in the 3A priority; Vg - age group; D - gender; P - occupation; J - field of education.

The calculation of the structure of jobseekers in comparison with the calculation of the second priority structure (248 formula) uses a new element - technical overall minimum of jobseekers (1 person), in formulas 261 and 262. This minimum is necessary as at certain levels of education (such as doctoral education) in the third employment priority may not be unemployed. In the absence of jobseekers, it is impossible to calculate the structure, without this technical supplement the model may not work.

$$VS_{\rm DPJt}^{3\rm A} = \left( VS_{\rm DPJt}^{2\rm B} - DMNpD_{\rm DPJt}^{2\rm B} \right) \vee 0 - DMNI_{\rm DPJt}^{2\rm B},$$
(263)

where

 $VS^{3A}_{DPJ}$  - number of vacancies in the 3A priority;  $VS^{2B}_{DPJ}$  - number of vacancies in the 2B priority; DMNpD<sup>2B</sup>\_{DPJ} - occupation of vacancies in the 2B priority by gender demand;
DMNI<sup>2B</sup><sub>DPI</sub> - changes of gender demand of the occupation of vacancies in the 2B priority; D - gender:

P - occupation;

J - field of education.

$$VSbD_{Pt}^{3A} = \sum_{n \in J} \sum_{i \in D} VS_{DPJt}^{3A \ in}, \qquad (264)$$

where

VSbDI<sup>3A</sup><sub>P</sub> - number of vacancies in the 3A priority without requirements for gender and education;

 $VS^{3A}_{DPJ}$  - number of vacancies in the 3A priority;

D - gender;

P - occupation;

J - field of education.

$$DMN_{VgDPJt}^{3A} = PPAAK^{3A} \times PPGAK^{3A} \times \times \begin{cases} DM_{VgDPJt}^{3A}, VSbDI_{PJt}^{3A} \times PS_{Vgt} \times DMdS_{Dt}^{3A} \times DMizS_{Jt}^{3A} \ge DM_{VgDPJt}^{3A} &, \quad (265) \\ VSbDI_{PJt}^{3A} \times PS_{Vgt} \times DMdS_{Dt}^{3A}, VSbDI_{PJt}^{3A} \times PS_{Vgt} \times DMdS_{Dt}^{3A} \times DMizS_{Jt}^{3A} < DM_{VgDPJt}^{3A} &, \quad (265) \end{cases}$$

where

DMN<sup>3A</sup><sub>VgDPJ</sub> - occupation of vacancies in the 3A priority; PPAAK<sup>3A</sup> - wages compliance coefficient of demand and supply in the 3A priority;

PPGAK<sup>3A</sup> - geographical compliance coefficient of demand and supply in the 3A priority;

DM<sup>3A</sup><sub>VgDPJ</sub> - number of jobseekers in the 3A priority;

VSbDI<sup>3A</sup><sub>P</sub> - number of vacancies in the 3A priority without requirements for gender and education;

 $PS_{Vg}$  - preferential age structure;

 $DMdS^{3A}_{D}$  - gender structure of jobseekers in the 3A priority;

 $DMizS^{3A}_{D}$  - education structure of jobseekers in the 3A priority;

Vg - age group;

D - gender;

P - occupation:

J - field of education.

Formulas 260 -265 show that the third priority sub-model of employment largely repeats the second priority sub-model of employment. The difference between them is related to observance of the education structure of jobseekers. Further the gender and education non-conformity balancing equations (266 - 277 formulas) are reflected.

$$VSdS_{Dt}^{3A} = \frac{\sum_{n \in P} \sum_{m \in J} VS_{DPJt}^{3A^{nm}}}{\sum_{i \in D} \sum_{n \in P} \sum_{m \in J} VS_{DPJt}^{3A^{inm}}},$$
(266)

where

 $VSdS^{3A}_{D}$  - gender structure of vacancies in the 3A priority;  $VS^{3A}_{DPJ}$  - number of vacancies in the 3A priority; D - gender; P - occupation; J - field of education.

$$VSizS_{Jt}^{3A} = \frac{\sum_{n \in P} \sum_{m \in D} VS_{DPJt}^{3A^{nm}}}{\sum_{i \in D} \sum_{n \in P} \sum_{m \in J} VS_{DPJt}^{3A^{inm}}},$$
(267)

where

VSizS<sup>3A</sup> J - education structure of vacancies in the 3A priority;
VS<sup>3A</sup> DPJ - number of vacancies in the 3A priority;
D - gender;
P - occupation;
J - field of education.

The equations of determination of the structure of vacancies (266 and 267 formulas) are analogical to the unified sub-models.

$$DMNbD_{Pt}^{3A} = \sum_{n \in J} \sum_{m \in Di \in Vg} DMN_{V_g DPJt}^{3A}, \qquad (268)$$

where

 $DMNbDI^{3A}_{P}$  - occupation of vacancies in the 3A priority without gender and education;  $DMN^{3A}_{VgDPJ}$  - occupation of vacancies in the 3A priority;

Vg - age group;

D - gender;

P - occupation;

J - field of education.

Formula 268 presents the calculation of the number of occupied vacancies, not reflecting the education and gender requirements. Then formula 269 presents the calculation of the number of occupied vacancies, which corresponds to the demand (by education and gender), and at the same time, it is subtracted from the initial number of vacancies, thus obtaining the number of vacancies for occupation of vacancies in accordance with the demand. In turn, formula 270 calculates the number of occupied vacancies, where the gender and education of employees do not correspond to the demand.

$$VSpA_{\rm DPJt}^{3\rm A} = \left(VS_{\rm DPJt}^{3\rm A} - DMNbDI_{\rm Pt}^{3\rm A} \times VSdS_{\rm Dt}^{3\rm A} \times VSizS_{\rm Jt}^{3\rm A}\right) \vee 0, \qquad (269)$$

where

VSpA<sup>3A</sup><sub>DPJ</sub> - number of vacancies after occupation of vacancies by gender and education demand in the 3A priority;

VS<sup>3A</sup><sub>DPJ</sub> - number of vacancies in the 3A priority;

DMNbDI<sup>3A</sup><sub>P</sub> - occupation of vacancies in the 3A priority without gender and education requirements;

 $VSdS^{3A}_{D}$  - gender structure of vacancies in the 3A priority;

VSizS<sup>3A</sup> J - education structure of vacancies in the 3A priority;

D - gender;

P - occupation;

J - field of education.

Calculating the number of vacancies after occupation of vacancies in accordance with the demand, the occupied vacancies (by occupations), multiplying them by the demanded gender and education structures, are subtracted from the initial number of vacancies. If the structures develop unallowable non-conformance of the demand and occupation, i.e., the occupation is larger than the demand, then in order to avoid negative (less than zero) number of vacancies, formula 269

minimum is set at zero level. Thus, formula 269 represents the number of vacancies after the appropriate occupation of vacancies by gender and education demand. The number of vacancies, which indicates unallowable non-conformance of the demand and occupation is analysed in formula 270:

$$DIN_{\rm DPJt}^{\rm 3A} = -\left(\left(VS_{\rm DPJt}^{\rm 3A} - DMNbDI_{\rm Pt}^{\rm 3A} \times VSdS_{\rm Dt}^{\rm 3A} \times VSizS_{\rm Jt}^{\rm 3A}\right) \wedge 0\right),\tag{270}$$

where

DIN<sup>3A</sup><sub>DPJ</sub> - noncompliance of gender and education demand of occupation of vacancies in 3A priority;

 $VS^{3A}_{DPJ}$  - number of vacancies in the 3A priority;

 $DMNbDI^{3A}_{P}$  - occupation of vacancies in the 3A priority without gender and education;

VSdS<sup>3A</sup><sub>D</sub> - gender structure of vacancies in the 3A priority;

 $VSizS^{3A_{I}}$  - education structure of vacancies in the 3A priority;

D - gender;

P - occupation;

J - field of education.

Formula 270 reflects non-conformance of the number of vacancies or occupation. This calculation is similar to formula 269, with the only difference that formula 269 applies the minimum restraint (zero), but in formula 270, the same level is taken as the maximum. In order to avoid the negative element, the sign is changed in formula 270.

By explaining the nature and significance of formulas 269 and 270, we can say that formula 269 reflects the number of remaining vacancies after the appropriate occupation, and formula 270 reflects vacancies, which are occupied by jobseekers but those vacancies do not reduce the number of free vacancies, taking into account the non-conformance of jobseekers and number of vacancies (by gender and education). Further, in formulas 274 and 275, the vacancies defined in formula 270 are distributed by the vacancies defined in formulac269 on the basis of the structure, calculation of which is presented in formulas 271 - 272.

$$VSdS 2_{Dt}^{3A} = \frac{\sum_{n \in P} \sum_{m \in J} VSp A_{DPJt}^{3A}{}^{nm}}{\sum_{i \in D} \sum_{n \in P} \sum_{m \in J} VSp A_{DPJt}^{3A}{}^{inm}},$$
(271)

where

 $VSdS2^{3A}$  D - age structure of vacancies in the 3A priority 2;

 $VSpA^{3A}_{DPJ}$  - number of vacancies after occupation of vacancies by gender and education demand in the 3A priority;

D - gender;

P - occupation;

J - field of education.

$$VSizS2_{Jt}^{3A} = \frac{\sum_{n \in P} \sum_{m \in D} VSpA_{DPJt}^{3A^{nm}}}{\sum_{i \in D} \sum_{n \in P} \sum_{m \in J} VSpA_{DPJt}^{3A^{inm}}},$$
(272)

where

VSizS2<sup>3A</sup><sub>J</sub> - education structure of vacancies in the 3A priority 2; VSpA<sup>3A</sup><sub>DPJ</sub> - number of vacancies after occupation of vacancies by gender and education demand in the 3A priority;

D - gender;

P - occupation;

J - field of education.

$$DMNbD2_{Pt}^{3A} = \sum_{n \in J} \sum_{m \in D} DIN_{DPJt}^{3A}^{mn} , \qquad (273)$$

where

DMNbDI2<sup>3A</sup><sub>P</sub> - noncompliance of gender and education demand of occupation of vacancies in the 3A priority without education and gender;

DIN<sup>3A</sup><sub>DPJ</sub> - noncompliance of gender and education demand of occupation of vacancies in the 3A priority;

D - gender;

P - occupation;

J - field of education.

$$VSpA2_{DPJt}^{3A} = \left( VSpA_{DPJt}^{3A} - DMNbDP_{Pt}^{3A} \times VSdS2_{Dt}^{3A} \times VSizS2_{Jt}^{3A} \right) \vee 0, \qquad (274)$$

where

VSpA2<sup>3A</sup><sub>DPJ</sub> - number of vacancies after the occupation of vacancies by remaining gender and education structure in the 3A priority;

VSpA<sup>3A</sup><sub>DPI</sub> - number of vacancies after occupation of vacancies by gender and education demand in 3A priority;

DMNbDI2<sup>3A</sup><sub>DPJ</sub> - noncompliance of gender and education demand of occupation of vacancies in 3A priority;

 $VSdS2^{3A}_{D}$  - gender structure of vacancies in the 3A priority;

 $VSizS2^{3A}$  - education structure of vacancies in the 3A priority;

D - gender;

P - occupation;

J - field of education.

...

$$DIN2_{\text{DPJt}}^{3\text{A}} = -\left(\left(VSpA_{\text{DPJt}}^{3\text{A}} - \text{DMNbDI2}_{\text{Pt}}^{3\text{A}} \times VSdS2_{\text{Dt}}^{3\text{A}} \times VSizS2_{\text{Jt}}^{3\text{A}}\right) \wedge 0\right),$$
(275)

where

DIN2<sup>3A</sup><sub>DPJ</sub> - noncompliance of gender and education demand of occupation of vacancies in the 3A priority 2;

VSpA<sup>3Å</sup><sub>DPI</sub> - number of vacancies after occupation of vacancies by gender and education demand in the 3A priority;

DMNbDI2<sup>3A</sup><sub>DPI</sub> - noncompliance of gender and education demand of occupation of vacancies in the 3A priority;

 $VSdS2^{3A}_{D}$  - gender structure of vacancies in the 3A priority;

 $VSizS2^{3A}$  J - education structure of vacancies in the 3A priority;

D - gender;

P - occupation;

J - field of education.

Analysing formulas 271 - 275, it is evident that they largely repeat formulas 266 - 270. These two groups provide the transition from job seekers to the number of vacancies and form the cycle, where jobseekers during employment reduce the number of free vacancies.

In the end of the cycle, if the model preserves the unallowable non-conformities of the labour demand and occupation of vacancies, they are divided in proportion to vacancies (the proportion observes the gender, education and occupation). This division, as well as the calculation of vacancies for the next priority are reflected by formulas 276 and 277.

$$AVSA_{DPJt}^{3A} = \frac{VSpA2_{DPJt}^{3A}}{\sum_{i\in D}\sum_{n\in P}\sum_{m\in J}VSpA2_{DPJt}^{3A}},$$
(276)

where

AVSA<sup>3A</sup><sub>DPI</sub> - structure of vacancies in the 3A priority;

VSpA2<sup>3A</sup><sub>DPJ</sub> - number of vacancies after occupation of vacancies by remaining gender and education structure in the 3A priority;

D - gender;

P - occupation;

J - field of education.

Formula 276 reflects the structure of vacancies, then formula 277 presents the proportional division of unallowable non-conformities of the occupation of vacancies:

$$VS_{DPJt}^{3B} = \left( VSpA2_{DPJt}^{3A} - AVSA_{DPJt}^{3A} \times \sum_{i \in D} \sum_{n \in P} \sum_{m \in J} DIN2_{DPJt}^{3A} \right) \vee 0, \qquad (277)$$

where

 $VS^{^{3B}}_{^{}DPJ}$  - number of vacancies in 3B priority;  $VSpA2^{^{3A}}_{^{}DPJ}$  - number of vacancies after occupation of vacancies by remaining gender and education structure in 3A priority;

AVSA<sup>3A</sup><sub>DPJ</sub> - structure of vacancies in 3A priority;

DIN2<sup>3A</sup><sub>DPJ</sub> - noncompliance of gender and education demand of occupation of vacancies in 3A priority 2;

D - gender;

P - occupation;

J - field of education.

Formula 277 reflects the number of vacancies for the next priority.

Figure 2.56 reflects the fourth priority sub-model of employment, in which jobseekers change their occupations in accordance with the market requirements, observing the education of jobseekers.



Figure 2.56. Sub-model of the employment of the fourth priority

Comparing the fourth priority sub-model (Figure 2.56) with the third priority sub-model (Figure 2.55), it is evident that they differ mainly just prior to occupation of vacancies. After occupation of vacancies the existing differences can be related only to the application of the different structure: in the third priority sub-model the gender and education structures are applied, but in the fourth priority sub-model - gender and occupation structures. The part of the model after the occupation of the vacancies is explained in detail in formulas 266 - 277, therefore, it is not described again.

The feature of the fourth priority sub-model is related to the fact that jobseekers change occupation in order to get a job. Change of occupation is restrained by the attained education, i.e., with the certain education it is possible to work in different occupations and, when it is necessary, to change occupation according to the attained education.

By implementing this principle in the model, Figure 2.56 shows that the number of jobseekers who have changed occupations (in the previous priorities it is the occupation of vacancies) are calculated in the beginning. This index reflects the number of jobseekers, which will occupy the vacant working places. In this phase jobseekers retain their previous occupations. This index is also important to calculate the number of jobseekers in accordance with the priority in the end of algorithm. Then the jobseekers are assigned occupations and they occupy vacancies in the demanded occupations (i.e., considering the occupational structure of vacancies). The equations of

the part of the fourth priority sub-model (before occupation of the vacancies) of employment are presented in formulas 278 - 283.

$$DMkS_{\text{DPJt}}^{4\text{A}} = \frac{\sum_{k \in Vg} DM_{Vg\text{DPJt}}^{4\text{A}}}{1 \vee \sum_{i \in D} \sum_{k \in Vg} \sum_{n \in P} \sum_{m \in J} DM_{Vg\text{DPJt}}^{4\text{A}}},$$
(278)

where

DMkS<sup>4A</sup><sub>DPJ</sub> - structure of jobseekers in the 4A priority;  $DM^{4A}_{VgDPJ}$  - number of jobseekers in the 4A priority; Vg - age group; D - gender; P - occupation; J - field of education.

Formula 278 reflects the structure of jobseekers in the 4A priority.

$$VSbDP_{Jt}^{4A} = \sum_{n \in P} \sum_{i \in D} VS_{DPJt}^{4A}^{in}, \qquad (279)$$

where

VSbDP<sup>4A</sup><sub>J</sub> - number of vacancies in the 4A priority without gender and occupational requirements;

VS<sup>4A</sup><sub>DPJ</sub> - number of vacancies in the 4A priority;

D - gender;

P - occupation;

J - field of education.

Formula 279 presents the calculation of the number of vacancies by educational levels without gender and occupational requirements.

$$PMDM_{VgDPJt}^{4A} = PPAAK^{4A} \times PPGAK^{4A} \times \times \begin{cases} DM_{VgDPJt}^{4A}, VSbDP_{PJt}^{4A} \times PS_{Vgt} \times DMkS_{Dt}^{4A} \ge DM_{VgDPJt}^{4A}, \\ VSbDP_{PJt}^{4A} \times PS_{Vgt} \times DMkS_{Dt}^{4A}, VSbDP_{PJt}^{4A} \times PS_{Vgt} \times DMkS_{Dt}^{4A} < DM_{VgDPJt}^{4A} \end{cases}$$

$$(280)$$

where

PMDM<sup>4A</sup><sub>VgDPJ</sub> - change of occupation in the 4A priority;

PPAAK<sup>4A</sup> - wage compliance coefficient of demand and supply in the 4A priority;

 $PPGAK^{4A}$  - geographical compliance coefficient of demand and supply in the interprising, priority;

DM<sup>4A</sup><sub>VgDPJ</sub> - number of jobseekers in the 4A priority;

VSbDP<sup>4A</sup><sub>J</sub> - number of vacancies in the 4A priority without gender and occupational requirements;

 $PS_{Vg}$  - preferential age structure; DMkS<sup>4A</sup><sub>DPJ</sub> - structure of jobseekers in the 4A priority;

Vg - age group;

D - gender;

P - occupation;

J - field of education.

Formula 280 presents the calculation of the number of jobseekers, which changed the occupation. Formula 280 is analogous to formulas 246, 251 and 265 that calculate the occupation of vacancies. However, formula 280 does not immediately reflect the occupation of vacancies, but reflect the number of jobseekers that occupy the vacancies with the previous occupations. Formula 281 analyses the number of jobseekers without previous occupations:

$$DMNbP_{VgDJt}^{4A} = \sum_{n \in P} PMDM_{VgDPJt}^{4A}, \qquad (281)$$

where

 $DMNbP^{4A}_{VgDJ}$  - occupation of vacancies in the 4A priority without occupations;  $PMDM^{4A}_{VgDPJ}$  - change of occupation in the 4A priority; Vg - age group; D - gender; P - occupation;

J - field of education.

$$VSpS2_{Pt}^{4A} = \frac{\sum_{n \in J} \sum_{m \in D} VS_{DPJt}^{4A^{nm}}}{\sum_{i \in D} \sum_{n \in P} \sum_{m \in J} VS_{DPJt}^{4A^{inm}}},$$
(282)

where

 $VSpS^{4A}_{P}$  - occupational structure of the vacancies in the 4A priority;  $VS^{4A}_{DPJ}$  - number of vacancies in the 4A priority; D - gender; P - occupation;

J - field of education.

Formula 282 reflects the occupational structure of vacancies.

$$DMN_{V_{g}\text{DPJt}}^{4A} = DMNbP_{V_{g}\text{DJt}}^{4A} \times VSpS_{P_{t}}^{4A}, \qquad (283)$$

where

 $DMN^{4A}_{VgDPJ}$  - occupation of vacancies in the 4A priority;  $DMNbP^{4A}_{VgDJ}$  - occupation of vacancies in the 4A priority without occupations;  $VSpS^{4A}_{P}$  - occupational structure of the vacancies in the 4A priority; Vg - age group; D - gender; P - occupation; J - field of education.

Formula 283 defines the occupation of vacancies, which is based on the occupational structure of vacancies and occupation of vacancies without occupations.

Figure 2.57 presents the fifth priority sub-model of employment, in which jobseekers change their occupations in accordance with the market requirements, observing the previous experience of jobseekers.



Figure 2.57. Sub-model of the employment of the fifth priority

Fifth priority sub-model (Figure 2.57) essentially differs from all previous models. It is based on the comparison of potential occupations of vacancies and jobseekers. It means that the vacant places by occupations are replaced by vacant places in occupations requiring the same (or similar) skills (similar occupations). At the same time, jobseekers by occupational groups move to similar occupations. If the "similar" vacancies coincide with similar occupations of job seekers and jobseekers occupy working places, the number of vacancies and jobseekers declines.

By implementing this principle, the fifth priority sub-model calculates the matrix of potentially new occupations for vacancies (on the basis of the compliance matrix of occupations), adjusts this matrix, taking into account the restraint of the number of vacancies, thus creating a new matrix of potentially new occupations. The matrix of potentially new occupations for vacancies reflects the occupations that can replace the requirements of the occupations of vacancies. On the basis of the number of jobseekers and matrix structure of vacancies the matrix of potentially new occupations for jobseekers is developed, which reflects occupations that can be changed by jobseekers while looking for a job. The structure of the change of occupation of jobseekers is defined by free vacancies.

The comparison of the occupational matrix is made in order to calculate the correspondence of jobseekers to the vacancies. The comparison also observes other factors, such as conformity of the supply and demand, wage and geography. Matrix comparison indicates the occupations of jobseekers both after and prior to the change of occupation. It is based on the calculation of the occupied vacancies and change of the occupation of jobseekers in order to get a job.

The equations of the fifth priority sub-model of employment are presented in formulas 284 - 295.

$$VSbDJ_{Pt}^{5A} = \sum_{n \in J} \sum_{i \in D} VS_{DPJt}^{5A}^{in} , \qquad (284)$$

where

 $VSbDJ_{P}^{5A}$  - number of vacancies in 5A priority without gender and occupational requirements;

VS<sup>5A</sup><sub>DPJ</sub> - number of vacancies in 5A priority;

D - gender;

P - occupation;

J - field of education.

$$PJPM_{PPt}^{5A} = VSbDJ_{Pt}^{5A} \times \frac{PAM_{PPt}}{1 \vee \sum_{i \in P2} PAM_{PP2t}^{i}}, \qquad (285)$$

where

PJPMv<sup>5A</sup><sub>PP</sub> -matrix of potential new occupations for vacancies in 5A priority;

 $VSbDJ_{P}^{5A}$  - number of vacancies in 5A priority without gender and occupational requirements;

PAM<sub>PP</sub> - compliance matrix of occupations;

P - occupation.

The number of vacancies in *i* priority in formula 285 by occupations (vector) is multiplied by the structure of the compliance matrix of occupations, as a result the matrix is obtained, which represents the potential divisions of vacancies by similar occupations. The structure of the compliance matrix of occupations is calculated by dividing the compliance matrix of occupations by the number of the similar occupations (vector, which is calculated by summing up the columns of the compliance matrix of occupations). This equation (formula 285) shows that the vacancies in occupations can be replaced by other occupations of vacancies, in the same ratio for all similar occupations. The united ratio approach may have disadvantages, for example, if jobseekers are divided by occupations in other ratios and the calculated division of vacancies does not correspond to jobseekers; in order to close this gap, the fifth priority sub-model of the employment is employed in the cycle up to the moment when the non-conformance of the division of vacancies and jobseekers is subjected to adjustment.

The potential division of vacancies by similar occupations, presented in formula 285, has one more disadvantage: in the result of the equal division, the number of vacancies in some vacancies may be increased more than it is accessible (i.e. more than the demand). The situation, when the occupation may employ more people than demanded, is unacceptable. From the economic perspective, the development of such situation can be explained in the following way: the vacancies from the similar occupations are added to the existing vacancies in the occupation, resulting in the exceeding increase of the number of vacancies in the occupation. In order to close this gap, the restraint of the number of vacancies was introduced into the sub-model, formula 286:

$$VSR_{Pt}^{5A} = \begin{cases} VSbDJ_{Pt}^{5A}, \sum_{i \in P1} PJPM_{P1Pt}^{5A}^{i} \ge VSbDJ_{Pt}^{5A} \\ \sum_{i \in P1} PJPM_{P1Pt}^{5A}^{i}, \sum_{i \in P1} PJPM_{P1Pt}^{5A}^{i} < VSbDJ_{Pt}^{5A}, \end{cases}$$
(286)

where

 $VSR^{5A}_{P}$  - restraint of the number of vacancies in 5A priority;  $VSbDJ^{5A}_{P}$  - number of vacancies in 5A priority without gender and education requirements;  $PJPMv^{5A}_{PP}$  - matrix of potential new occupations for vacancies in 5A priority; P - occupation.

Restraint of the number of vacancies sums up the lines of the compliance matrix of occupation and compares them with the number of vacancies by occupations (equal to the sum of column of the compliance matrix of occupation); from these elements the smaller is chosen. This ensures that the occupation will not employ more employees than demanded.

The index "restraint of the number of vacancies" represents the number of vacancies by occupations, which can be occupied by jobseekers in the model. This index may be less than the number of vacancies in i priority. Not to lose the vacancies, the fifth priority sub-model of the employment is repeated in the cycle while regulating the non-conformance of the division of vacancies and jobseekers.

The restraint structure of the number of vacancies or the division structure of vacancies by related occupations is calculated in formula 287:

$$VSSS_{\text{PPt}}^{5A} = \begin{cases} \left[\frac{PJPM_{\text{PPt}}^{5A}}{\sum PJPM_{\text{PIPt}}^{5A}}\right]^{T}, \left[\frac{PJPM_{\text{PPt}}^{5A}}{\sum PJPM_{\text{PIPt}}^{5A}}\right]^{T} > 0 \\ 0, \left[\frac{PJPM_{\text{PPt}}^{5A}}{\sum PJPM_{\text{PIPt}}^{5A}}\right]^{T} \le 0 \\ 0, \left[\frac{PJPM_{\text{PPt}}^{5A}}{\sum PJPM_{\text{PIPt}}^{5A}}\right]^{T} \le 0 \\ 0, \frac{1}{0} \end{cases}$$
(287)

where

 $VSSS^{5A}_{PP}$  - division structure of vacancies by related occupations in 5A priority; PJPMv<sup>5A</sup><sub>PP</sub> - matrix of potential new occupations for vacancies in 5A priority; P - occupation.

Division structure of vacancies is calculated on the basis of the matrix of potential new occupations. The specific type of equation (formula 287) is related to the technical characteristics of the model, and shows that if it is not possible to calculate the structure (the resulting number of vacancies in the occupation is equal to zero) or the calculation is not logical (less than zero), all structural elements are assigned the value of zero, otherwise the structure is calculated in a classical way. Matrix transposition is related to the calculation of the structure by the matrix lines, but then the matrix structure of the column section is required.

On the basis of the division structure of vacancies and restraint of the number of vacancies, the matrix of the potential new occupations is calculated for the vacancies, formula 288:

$$JPMv_{\rm PPt}^{\rm 5A} = VSSS_{\rm PPt}^{\rm 5A} \times VSR_{\rm Pt}^{\rm 5A}, \qquad (288)$$

where

JPMv<sup>5A</sup><sub>PP</sub> - matrix of potential new occupations for vacancies in 5A priority; VSSS<sup>5A</sup><sub>PP</sub> - division structure of vacancies by related occupations in 5A priority; VSR<sup>5A</sup><sub>P</sub> - restraint of the number of vacancies in 5A priority; P - occupation.

Matrix of potential new occupations for vacancies reflects the occupations that may have the changed occupational demands of the vacancies. The sum of this matrix by lines reflects initial requirements of the occupations of vacancies, but the sum by columns reflects the equivalent potential requirements, while observing the interdependence of occupations.

After the calculation of the matrix of potential new occupations of vacancies, the matrix of potential new occupations for jobseekers is calculated, formula 289:

$$JPMdm_{\rm PPt}^{5A} = \sum_{i \in D} \sum_{k \in Vg} \sum_{m \in J} DM_{VgDPJt}^{5A} \stackrel{ikm}{\times} \times \frac{JPMv_{\rm PPt}^{5A}}{\sum_{i \in P2} JPM_{\rm PP2t}^{5A}},$$
(289)

where

JPMdm<sup>5A</sup><sub>PP</sub> - matrix of possible new occupations for the number of jobseekers in 5A priority;

 $DM^{5A}_{VgDPJ}$  - number of jobseekers in 5A priority;

 $JPMv_{PP}^{5A}$  - matrix of possible new occupations for vacancies in 5A priority;

Vg - age group;

D - gender;

P - occupation;

J - field of education.

In the calculation of the matrix of potential new occupations of jobseekers (formula 289), the jobseekers are summed up to the occupational level (vector), the matrix is formed by the correlation structure, which is based on the occupational structure of vacancies, i.e., the structure of the change of occupation of jobseekers is defined by free vacancies.

The developed matrixes of potential new occupations are compared in formula 290:

$$PMS_{PPt}^{5A} = PPAAK^{5A} \times PPGAK^{5A} \times \begin{cases} JPMdm_{Ppt}^{5A}, JPMv_{Ppt}^{5A} > JPMdm_{Ppt}^{5A} \\ JPMv_{Ppt}^{5A}, JPMv_{Ppt}^{5A} \le JPMdm_{Ppt}^{5A} \end{cases},$$
(290)

where

PMS<sup>5A</sup><sub>P</sub> - comparison of occupations matrixes in the 5A priority;

PPAAK<sup>5A</sup> - wage compliance coefficient of demand and supply in the 5A priority;

PPGAK<sup>5A</sup> - geographical compliance coefficient of demand and supply in the 5A priority;

JPMv<sup>5A</sup><sub>PP</sub> - matrix of possible new occupations for vacancies in the 5A priority;

JPMdm<sup>5A</sup><sub>PP</sub> - matrix of potential new occupations for the number of jobseekers in the 5A priority;

P - occupation.

The comparison of the occupational matrix is made in order to calculate the correspondence of jobseekers to vacancies. Formula 290 shows that it also observes other factors, such as wage and geographical conformance of supply and demand. The resulting matrix reflects which occupations are replaced by jobseekers in order to get a job. These indexes in formulas 290 and 291 are

distinguished separately, and then they are used for decline in the number of free vacancies and jobseekers.

$$PMSpe_{Pt}^{5A} = \sum_{i \in P2} PMS_{PP2t}^{5A \ i} , \qquad (291)$$

where

 $PMSpe^{5A}_{P}$  - occupations of jobseekers after the change of occupation in the 5A priority;  $PMS^{5A}_{PP}$  - comparison of occupations matrixes in the 5A priority; P - occupation.

$$PMSpi_{Pt}^{5A} = \sum_{i \in P2} PMS_{PP2t}^{5A^{i}}, \qquad (292)$$

where

 $PMSpi_{P}^{5A}$  - occupations of jobseekers before the change of occupation in the 5A priority;  $PMS_{PP}^{5A}$  - comparison of occupations matrixes in the 5A priority; P - occupation.

As formulas 291 and 292 show, they differ by summing up the matrix elements by rows and columns.

The occupations of jobseekers after the change of occupation, i.e. the occupations of occupied vacancies. The vacancies are analysed in details in the sub-model, including the requirements of vacancies for the education and gender. The requirements of occupied vacancies in relation to the education and gender are calculated according to the structure of vacancies, formula 293:

$$DMNbV_{DPJt}^{5A} = PMSpe_{Pt}^{5A} \times AVSA_{DPJt}^{5A}, \qquad (293)$$

where

DMNbV<sup>5A</sup><sub>DPJ</sub> - occupation of vacancies in the 5A priority without age; PMSpe<sup>5A</sup><sub>P</sub> - occupations of jobseekers after the change of occupation in the 5A priority; AVSA<sup>5A</sup><sub>DPJ</sub> - structure of vacancies in the 5A priority; D - gender;

P - occupation;

J - field of education.

The index "occupation of vacancies in *i* priority without age" reflects the quantity and types of vacancies that are occupied in the current priority. Then this index is used by calculating the number of free vacancies in the next priority. The structure of vacancies is calculated by the unified equation (formula 276), so it is not re-presented again.

Formula 294 reflects the calculation of the parameter "change of occupation":

$$PMDM_{V_gDPJt}^{5A} = PMSpi_{Pt}^{5A} \times DMkS_{V_gDPJt}^{5A}, \qquad (294)$$

where

PMDM<sup>5A</sup><sub>VgDPJ</sub> - change of occupation in the 5A priority;
PMSpi<sup>5A</sup><sub>P</sub> - occupations of jobseekers before the change of occupation in the 5A priority;
DMkS<sup>5A</sup><sub>VgDPJ</sub> - structure of jobseekers in the 5A priority;
Vg - age group;
D - gender;
P - occupation;
J - field of education.

The index "change of occupation in i priority" reflects the quantity and types of jobseekers (including by the occupations) which have found jobs and have been employed. On the basis of this index the "structure of jobseekers in i priority 2" has been calculated, which reflects the age and education structure of jobseekers (i.e. new employees). Structures of jobseekers are calculated on the basis of the unified equations, therefore, are not reflected here again. The calculation of the second structure is related to the fact that in new working places the structure of employees should not correspond to the structure of jobseekers, but to the structure, which is characteristic to jobseekers with new occupation. The index "change of occupation in i priority" is used, by reducing the number of jobseekers.

The number of jobseekers who occupy working places is calculated on the basis of the structures of new employees and jobseekers (after the change of occupation), formula 295:

$$DMN_{VgDPJt}^{5A} = PMSpe_{Pt}^{5A} \times DMkS2_{VgDPJt}^{5A}, \qquad (295)$$

where

 $DMN^{5A}_{VgDPJ}$  - occupation of vacancies in the 5A priority;  $PMSpe^{5A}_{P}$  - occupations of jobseekers after the change of occupation in the 5A priority;  $DMkS2^{5A}_{VgDPJ}$  - structure of jobseekers in the 5A priority 2; Vg - age group; D - gender; P - occupation; J - field of education.

The index "occupation of vacancies in i priority" is used by increasing the number of employed.

The fifth priority sub-model of employment is repeated in the cycle until the moment, when non-compliance of vacancies distribution and jobseekers is subject of regulations.

In the explanation of the fifth priority sub-model of employment the occupational compliance matrix was mentioned. The calculation of the occupational compliance matrix is not related to the sub-model. Considering that it consists of three equations, it is not appropriate to distinguish a separate subsection for it. The essence of the calculation of the occupational compliance matrix is presented below.

The occupational compliance matrix is based on the education compliance matrix and reflects the interchangeable occupations. In order to obtain the occupational compliance matrix from the education compliance matrix, in the first step the integrated education compliance matrix of the levels of education is developed, formula 296:

$$AIAM_{\rm PJt} = 1 \wedge \sum_{i \in L} IAM_{\rm PJt}^{i} , \qquad (296)$$

where

AIAM<sub>PJ</sub> - integrated education compliance matrix;

 $IAM^{i}_{PJ}$  - education compliance matrix of *i* level of education;

P - occupation;

J - field of education;

L - level of education.

As formula 296 shows, one matrix reflects all connections of the occupational-educational field of all levels of education. The maximum constraint 1 of formula 296 has been introduced in order to avoid duplication connections of the occupational-educational field from the different levels of education in the integrated matrix.

At the next stage the integrated education compliance matrix is transposed, formula 297:

$$TAIAM_{\rm JPt} = \left[AIAM_{\rm PJt}\right]^T, \tag{297}$$

where

TAIAM<sub>JP</sub> - transposition of integrated education compliance matrix;

AIAM<sub>PJ</sub> - integrated education compliance matrix;

P - occupation;

J - field of education.

multiplying the transposed education compliance matrix with the integrated education compliance matrix, the occupational compliance matrix is obtained, formula 298:

$$PAM_{\rm PPt} = 1 \wedge ([TAIAM_{\rm JPt}] \times [AIAM_{\rm PJt}]), \qquad (298)$$

where

PAM<sub>PP</sub> - occupational compliance matrix; TAIAM<sub>JP</sub> - transposition of integrated education compliance matrix; AIAM<sub>PJ</sub> - integrated education compliance matrix; P - occupation; J - field of education.

The maximum restraint is also applied to occupational compliance matrix.

After employment, all the fifth priority sub-model vacancies that may be occupied by jobseekers are occupied. As stated above, there is an increase of the number of employed (formulas 198 and 208) and decline in the number of jobseekers (formulas 197 and 207). Simultaneously, there is a decline in the number of free vacancies, which has not been previously described. Figure 2.58 reflects a decline in the number of free vacancies in the conceptual form.



Figure 2.58. Algorithm of decline in the number of vacancies

After the employment the five priorities in the sub-model have the element of the number of free vacancies that is marked as "number of vacancies in the 6 priority". comparing it to the initial number of vacancies (before the employment priorities), the number of occupied vacancies can be determined. Considering that vacancies may be occupied in accordance with the demand and without regard to the demand requirements in relation to gender or education, the occupation of

vacancies (occupation of working places) is divided into two parts: occupation of working places according to vacancies and occupation of working places according to occupation of vacancies. Both of these indexes are used for decline in the number of vacancies along with the occupation of working places (see 215 formula). Occupation of working places in accordance with the vacancies directly reduces the number of vacancies. Occupation of working places in accordance with the occupation of vacancies (with the non-conformity to gender or education) may lead to inappropriate decline in the number of vacancies, including the formation of a negative number of vacancies. To solve this problem, an algorithm of decline in the number of vacancies has calculated the index "remaining vacancies", which reflects the vacancies that remained after the occupation of working places according to vacancies. On the basis of the remaining vacancies, the structure of remaining vacancies is calculated, which ensures the appropriate division of the occupation of working places by fields of education and gender in accordance with occupations.

The equations of the decline in the number of vacancies are presented in formulas 299 - 302.

$$DVIX_{\rm DPJt} = \begin{cases} VS_{\rm DPJt} - VS_{\rm DPJt}^{6}, VS_{\rm DPJt} - VS_{\rm DPJt}^{6} > 0\\ VS_{\rm DPJt}, VS_{\rm DPJt} - VS_{\rm DPJt}^{6} \le 0 \end{cases},$$
(299)

where

 $\ensuremath{\text{DVIX}}_{\ensuremath{\text{DPJ}}}$  - the working place is being occupied in accordance with vacancies;

VS<sub>DPJ</sub> - number of vacancies;

VS<sup>6</sup><sub>DPJ</sub> - number of vacancies in the 6 priority;

D - gender;

P - occupation;

J - field of education.

Comparing the number of vacancies before employment priorities with the number of vacancies after the fifth priority, in formula 299 the occupation of working places (occupation of vacancies) is determined. Mathematically, it is positive (greater than zero) difference of constituent elements. Negative difference (less than zero) of these elements makes up an inappropriate occupation of vacancies, formula 300.

$$DVIN_{\rm DPJt} = -\left(0 \wedge \left(VS_{\rm DPJt} - VS_{\rm DPJt}^{6}\right)\right),\tag{300}$$

where

 $DVIN_{DPJ}$  - the working place is occupied in accordance with occupation of vacancy;  $VS_{DPJ}$  - number of vacancies;  $VS_{DPJ}^{6}$  - number of vacancies in the 6 priority; D - gender; P - occupation; J - field of education.

The gender and fields of education do not correspond to the requirements of vacancies in the inappropriate occupation of vacancies, while observing the constructing logics of the model. But the requirements by occupations are observed. Considering that mathematically, this index is negative (less than zero), the sign has been changed in formula 300.

$$AVS_{\rm DPJt} = VS_{\rm DPJt} - DVIX_{\rm DPJt}, \qquad (301)$$

where AVS<sub>DPJ</sub> - remaining vacancies; VS<sub>DPJ</sub> - number of vacancies; DVIX<sub>DPJ</sub> - the working place is occupied in accordance with vacancies; D - gender; P - occupation; J - field of education.

The remaining vacancies are calculated in order to obtain the structure of remaining vacancies, which provides the occupation of working places by occupations in accordance with an appropriate division by fields of education and gender.

$$AVSA_{\text{DPJt}} = \frac{AVS_{\text{DPJt}}}{1 \vee \sum_{k \in D} \sum_{m \in J} AVS_{\text{DPJt}}} M_{m \in J},$$
(302)

where

AVSA<sub>DPJ</sub> - structure of remaining vacancies; AVS<sub>DPJ</sub> - remaining vacancies; D - gender; P - occupation; J - field of education.

the standard equation is used for the calculation of structure of remaining vacancies, formula 302.

Figure 2.59 reflects the sixth priority sub-model of employment, in which the employed change their occupations in order to improve their financial situation (increase the amount of salaries), observing the available free working places (vacancies) in the labour market.



Figure 2.59. Sub-model of the employment of the sixth priority

Comparing Figure 2.59 (sixth priority sub-model of employment) with Figure 2.57 (fifth priority sub-model), it is evident that they are almost indistinguishable. The sixth priority do not apply the wage compliance index of the supply and demand, and beyond the figure borders the calculation of the technical elements (indexes) has remained, such as the calculation of the structure. The sixth priority sub-model of employment is largely repeating the fifth priority sub-model. The difference is related to the fact that in the fifth priority occupations are changed by jobseekers, but in the sixth priority – by employed. Taking into account the equivalence of the priority sub-models, their content is not re-explained again.

However, despite the equivalence of sub-models, separate sub-model equations are different. They include the comparison equation of the occupational matrix (290 formula), in the sixth priority the wage compliance index is not observed, the matrix of potential new occupations of vacancies (285 formula) is not based on the compliance matrix of occupations, but on the matrix of attractive occupations for the occupational mobility; matrix of potential new occupations for employed (289 formula) observes the occupational mobility index and wage relations in the related occupations. The different equations of the sixth priority are reflected in formulas 303 -305.

$$PMS_{PPt}^{6} = PPGAK^{6} \times \begin{cases} JPMdm_{PPt}^{6}, JPMv_{PPt}^{6} > JPMdm_{PPt}^{6} \\ JPMv_{PPt}^{6}, JPMv_{PPt}^{6} \le JPMdm_{PPt}^{6} \end{cases},$$
(303)

where

 $PMS^{6}_{P}$  - comparison of occupational matrix in the 6 priority;  $PPGAK^{6}$  - geographical compliance coefficient of demand and supply in the 6 priority;  $JPMv^{6}_{PP}$  - matrix of possible new occupations for vacancies in the 6 priority;  $JPMdm^{6}_{PP}$  - matrix of possible new occupations for employed in the 6 priority; P - occupation.

In the sixth priority the occupation matrix in the comparison equation (formula 303) does not have the wage compliance coefficient.

$$PJPM_{PPt}^{5A} = \begin{cases} VSbDJ_{Pt}^{5A} \times \frac{PPMPM_{PPt}}{\sum_{i \in P2} PPMPM_{PP2t}^{i}}, VSbDJ_{Pt}^{5A} \times \frac{PPMPM_{PPt}}{\sum_{i \in P2} PPMPM_{PP2t}^{i}} \ge 0\\ 0, VSbDJ_{Pt}^{5A} \times \frac{PPMPM_{PPt}}{\sum_{i \in P2} PPMPM_{PP2t}^{i}} \le 0\\ 0, \frac{1}{0} \end{cases}, \quad (304)$$

where

 $PJPMv_{PP}^{5A}$  - matrix of potential new occupations for vacancies in the 6 priority; VSbDJ<sub>P</sub><sup>5A</sup> - number of vacancies by occupations in the 6 priority; PPMPM<sub>PP</sub> - matrix of attractive occupations for the occupational mobility; P - occupation.

The calculation of the matrix of potential new occupations of vacancies (formula 304) is based on the attractive occupational matrix for the occupational mobility. This matrix indicates the attractive occupations in accordance with the level of salaries in different occupations. More details about the nature of the matrix and its formation are presented in the labour prise sub-model.

$$JPMdm_{PPt}^{5A} = PMK \times (1 - AAM_{PPt}) \times \begin{cases} N_{Pt} \times \frac{JPMv_{PPt}^{6}}{\sum_{i \in P2} JPM_{PP2t}^{6-i}}, N_{Pt} \times \frac{JPMv_{PPt}^{6-i}}{\sum_{i \in P2} JPM_{PP2t}^{6-i}} > 0 \\ 0, N_{Pt} \times \frac{JPMv_{PPt}^{6-i}}{\sum_{i \in P2} JPM_{PP2t}^{6-i}} \le 0 \\ 0, \frac{1}{0} \end{cases}, \quad (305)$$

where

JPMdm<sup>6</sup><sub>PP</sub> - matrix of possible new occupations for employed in 6 priority; PMK - index of occupational mobility;

 $AAM_{PP}$  - wage compliance matrix;

AAMpp - wage compnance matrix,

 $N_{\text{P}}$  - number of employed by occupations;

JPM $v_{PP}^{6}$  - matrix of possible new occupations for vacancies in the 6 priority; P - occupation.

The matrix of the potential new occupations of employed additionally observes the occupational mobility index (i.e. the certain proportion of employees is ready to change occupation in order to get a better job), and the wage compliance matrix, which nature and formation are explained in the labour price sub-model.

The sixth priority sub-model of employment is the last part of the labour market sub-model, after which the synchronization algorithm and wage formation sub-models are investigated.

### 2.3.1.6. Synchronization algorithm

The need for the synchronization algorithm is based on the simulated object specifics: labour can change the age, level and field of education, occupation and economic activity status at the same time. These changes are divided into two groups for the economically active population: job seekers and employed, who are additionally interacting with one another and can change their employment status. Analytically it is not possible to predict the total result of the impact of several factors on the simulated index. But, on the other hand, impossible results can be determined analytically and which contravene to common sense. Such results may include the negative number (less than zero) of jobseekers or employed. Accordingly, the impact of influencing factors is required to be restrained in order to prevent excess of the bounds of common sense by total result. It is ensured by the flow synchronization algorithm, or, simpler, - synchronization algorithm.

It is important to explain why the model can develop illogical results, i.e., the number of people can be negative. The flows are based on the structure of the index of the current period. But the flows act in the end of the current period. If the simple model, with one flow, is analysed, then the situation does not cause any problems. In case of multiple flows, pre-formed flows change the index structure, which causes the disparities of the index and flow, and can lead to logically unreasonable results.

Implementing the model in practice, this theoretical situation is the following: with the decline in the number of working places in a certain occupational and educational filed, the situation may arise that there is no employed with a combination of such occupation and education. This situation may arise due to the non-conformance of the model assumptions and statistics: the model provides that it is possible to obtain an occupation and find an appropriate job only with a certain education (or education groups), but the statistics sometimes shows that occupations are not related to education. But, despite this fact, it is necessary to reduce the number of employed. In order to avoid logically unreasonable results in such situations, the synchronization algorithm complements the existing flows.

In the labour analysis sub-model of the labour supply module this situation is solved by introducing the element "maximum of changes", and regulating the flows in accordance with the structure of the maximum of changes. The synchronization algorithm is based on different principles in this sub-section. First of all, the index number of employed is predicted in the next period, and, further, while predicting the non-conformance (negative number of employees); a negative number is applied to the correction, i.e., not changing the values and structure of the flows. Synchronization algorithm is presented in Figure 2.60.



Figure 2.60. Synchronization algorithm

Synchronization algorithm is based on the forecasts of the number of jobseekers and employed. They are calculated by taking into account the appropriate stocks (jobseekers or employed), as well as all flows.

The forecasts of the number of jobseekers and employed determine the forecasted nonconformities (negative value). Forecasts are compared in the element "correction of the number of jobseekers and employed", in case of non-conformity; the inappropriate result is transferred from one group to another, provided that this will not cause an inappropriate result in the second group. This reflects that, if the required employees lose their jobs after planning of the flows, then the provided action is not carried out in the group of employees (as was expected), but in the group of jobseekers.

If the correction of the number of jobseekers and employed does not ensure the prevention of non-conformity (it is checked in the element "required correction of employed structure"), the age group correction of the employed is carried out. The age group correction of employed includes two stages, looking for appropriate employees in the younger and older age groups. This points out that in case after the flow planning the required employed persons may change the age group, the algorithm with the provided actions transmits into another age group.

Synchronization algorithm equations are presented in formulas 306 - 312.

Formulas 306 and 307 reflect the forecasts of the number of jobseekers and employed.

$$DMSP_{VgDPJt} = DM_{VgDPJt} + DMP_{VgDPJt} - PMDM_{VgDPJt} - DMN_{VgDPJt} + NDM_{VgDPJt} - DMS_{VgDPJt} - DMVS_{(Vg-1)DPJt} + DMVS_{VgDPJt} + KOR_{VgDPJt}, '$$

$$Vg = \begin{cases} \{20_{24}, \dots, 65_{69}, 70_{74}\}, Vg \in NDMvS_{(Vg-1)DPJ} \lor DMVS_{VgDPJ} \\ \{15_{19}, 20_{24}, \dots, 65_{69}, 70_{74}\}, Vg \in DMP_{VgDPJ} \lor PMDM_{VgDPJ} \lor DMN_{VgDPJ} \lor DMS_{VgDPJ}, (306) \end{cases}$$

where

 $\begin{array}{l} DMSP_{VgDPJ} \mbox{-} forecast of the number of jobseekers;} \\ DM_{VgDPJ} \mbox{-} number of jobseekers;} \\ DMP_{VgDPJ} \mbox{-} increase of the number of jobseekers;} \\ PMDM_{VgDPJ} \mbox{-} change of occupations in order to get a job;} \\ DMN_{VgDPJ} \mbox{-} move from job search to employment;} \\ NDM_{VgDPJ} \mbox{-} move from employment to job search;} \\ DMS_{VgDPJ} \mbox{-} decline in the number of jobseekers;} \\ DMvS_{VgDPJ} \mbox{-} change of age structure of jobseekers;} \\ Vg \mbox{-} age group; \\ D \mbox{-} gender;} \end{array}$ 

- P occupation;
- J field of education.

$$NSP_{VgDPJt} = N_{VgDPJt} + DMN_{VgDPJt} - NPM_{VgDPJt} - NDM_{VgDPJt} + + PMN_{VgDPJt} + NP_{VgDPJt} - NS_{VgDPJt} - NvS_{(Vg-1)DPJt} + NvS_{VgDPJt}, 'Vg = \begin{cases} \{20_{24}, \dots, 65_{69}, 70_{74}\}, Vg \in NvS_{(Vg-1)DPJ} \lor NvS_{VgDPJ} \\ \{15_{19}, 20_{24}, \dots, 65_{69}, 70_{74}\}, Vg \in DMN_{VgDPJ} \lor NPM_{VgDPJ} \lor , \\ \lor NDM_{VgDPJ} \lor PMN_{VgDPJ} \lor NS_{VgDPJ} \end{cases}$$
(307)

where

NSP<sub>VgDPJ</sub> - forecast of the number of employed;

 $N_{VgDPJ}$  - number of employed;

 $\begin{array}{l} DMN_{VgDPJ} - \text{move from job search to employment;} \\ NPM_{VgDPJ} - \text{move from employment to job search;} \\ NDM_{VgDPJ} - \text{move from employment to job search;} \\ PMN_{VgDPJ} - \text{employment after the change of occupation;} \\ NS_{VgDPJ} - \text{decline in the number of jobseekers due to the change of age structure;} \\ NP_{VgDPJ} - \text{decline in the number of jobseekers;} \\ NvS_{VgDPJ} - \text{change of age structure of employed;} \\ Vg - age group; \\ D - gender; \\ P - \text{occupation;} \end{array}$ 

J - field of education.

The forecasts of the number of jobseekers and employed reflect related stocks and flow amounts for the current period. Specifics of the system dynamics method, shows that the stocks reflect the current state of the period, but the flows - the changes of the period, allowing to calculate in the current period the condition of repository for the next period, which is realized in formulas 306 and 307.

On the basis of the forecasts of the number of jobseekers and employed, the correction of the number of jobseekers and employed is carried out, formula 308:

$$DMNKOR_{VgDPJt} = \frac{\left(NSP_{VgDPJt} < 0 \oplus \left(NSP_{VgDPJt} + DMSP_{VgDPJt}\right) \ge 0\right) \Longrightarrow NSP_{VgDPJt} \land 0 - \left(DMSP_{VgDPJt} < 0 \oplus \left(DMSP_{VgDPJt} + NSP_{VgDPJt}\right) \ge 0\right) \Longrightarrow DMSP_{VgDPJt} \land 0, (308)$$

where  $DMNKOR_{VgDPJ}$  - correction of the number of jobseekers and employed;  $NSP_{VgDPJ}$  - forecast of the number of employed;  $DMSP_{VgDPJ}$  - forecast of the number of jobseekers; Vg - age group; D - gender; P - occupation; J - field of education.

The calculation of the correction of the number of jobseekers and employed consists of two parts: the correction of the number of jobseekers is subtracted from the correction of the number of employed. Both parts of correction are calculated in a similar manner: if the negative value is forecasted, and, at the same time, the total amount of jobseekers and employed is greater or equals to zero, then the correction, which coincides with the expected negative value, is possible.

After the calculation of the correction of the number of jobseekers and employed it is checked whether this correction ensures the elimination of non-conformance, formula 309:

$$NNKOR_{\rm VgDPJt} = \left(NSP_{\rm VgDPJt} - DMNKOR_{\rm VgDPJt}\right) \wedge 0, \qquad (309)$$

where

NNKOR<sub>VgDPJ</sub> - required correction of the structure of employed; NSP<sub>VgDPJ</sub> - forecast of the number of employed; DMNKOR<sub>VgDPJ</sub> - correction of the number of jobseekers and employed; Vg - age group; D - gender; P - occupation; J - field of education.

The required correction of employed structure adjustment is calculated as a negative value, by subtracting the correction of the number of jobseekers and employed from the forecast of the number of employed. If the correction of the number of jobseekers and employed ensures the prevention of non-conformance, the result is equal to zero, and the further correction is not required. However, when the further correction is required, the correction of the age group of employed is made, formula 310:

$$V1NKOR_{\rm VgDPJt} = NNKOR_{\rm (Vg-1)DPJt}, \qquad (310)$$

where  $V1NKOR_{VgDPJ}$  - age group correction 1;  $NNKOR_{VgDPJ}$  - required correction of the structure of employed; Vg - age group; D - gender; P - occupation; J - field of education.

In the correction of the first age group for the required correction of employed structure the age group is changed, assigning the new group to the previous age group, which is one stage

younger than in the analysed group. This reflects the fact that the employed can become older at the forecasting stage, but in order to provide a positive number of employed, it will be returned in accordance with the flow synchronization.

After the first age group correction it is checked whether there is a need for further correction formula 311:

$$PNNKOR_{\rm VgDPJt} = \left(NSP_{\rm VgDPJt} - DMNKOR_{\rm VgDPJt} + V1NKOR_{\rm VgDPJt} - NNKOR_{\rm VgDPJt}\right) \wedge 0, \qquad (311)$$

where

PNNKOR<sub>VgDPJ</sub> - additionally required correction of employed structure;
NSP<sub>VgDPJ</sub> - forecast of the number of employed;
DMNKOR<sub>VgDPJ</sub> - correction of the number of jobseekers and employed;
V1NKOR<sub>VgDPJ</sub> - age group correction1;
NNKOR<sub>VgDPJ</sub> - required correction of employment structure;
Vg - age group;
D - gender;
P - occupation;
J - field of education.

By calculating the additionally required correction of the employed structure, the first correction of the number of jobseekers and employed, as well as employed age group is observed. If at this stage the non-conformities are detected, they are liquidated in the second age group correction, formula 312:

$$V2NKOR_{VgDPJt} = PNNKOR_{(Vg+2)DPJt},$$
(312)

where

V2NKOR<sub>VgDPJ</sub> - age group correction 2; PNNKOR<sub>VgDPJ</sub> - additionally required correction of employed structure; Vg - age group; D - gender; P - occupation; J - field of education.

In the second age group correction the age group is changed, by assigning a new group to the former age group, which is one step older than the initial group (up to age group correction). This reflects the fact that the flows may become older at the forecasting stage, but to provide a positive number of employed, it should be returned in accordance with the number of employed.

The existing flows on the basis of the synchronization algorithm are investigated along with all the other sub-model flows. Then the wage sub-model is investigated.

# 2.3.2. Wage sub-model

The calculation of wages in SDM model is based on the productivity growth considering the mutual relations of vacancies and the number of jobseekers. Wage sub-model is presented in Figure 2.61.



Figure 2.61. Wage sub-model

Wage sub-model reflects that the labour price (wages) is formed by balanced wages in longterm period, as well as it changes the modification index, which reflects the labour surplus or shortage and minimum wage. Wages can be analysed by occupational groups, levels of education and sectors (algorithm of relation of wage and sector productivity). Relation algorithm of wage and sector links the productivity of balanced wage and sectors in a long-term period.

The labour price sub-model equations are presented in formulas 312 - 333.

$$APT_{\rm Pt} = \frac{VS_{\rm Pt}}{1 \vee \left(N_{\rm Pt} + DM_{\rm Pt}\right)},\tag{313}$$

where

 $APT_P$  - wage growth index in accordance with the shortage of employees in occupations;  $VS_P$  - number of vacancies by occupations;  $N_P$  - number of employed by occupations;  $DM_P$  - number of jobseekers by occupations; P - occupation.

Wage growth index in accordance with the shortage of employees in occupations reflects the relation of the number of vacancies and the economically active population by occupations. The more vacancies exist, the more wages in the occupations grow. 312 formula calculates the economically active population by summing the number of employed and jobseekers. A minimum has been applied to economically active population by occupations - 1 person, this minimum is necessary to ensure the functionality of the model, taking into account that the statistical data reflects the fact that not all occupations involve economically active population.

$$ASB_{\rm Pt} = \frac{DM_{\rm Pt}}{1 \vee \left(N_{\rm Pt} + DM_{\rm Pt}\right)},\tag{314}$$

where

 $ASB_P$  - wage reduction index in accordance with a number of unemployed in occupations;  $DM_P$  - number of jobseekers by occupations;

 $N_P$  - number of employed by occupations;

P - occupation.

Wage reduction index in accordance with the number of unemployed in occupations reflects the ratio of the total number of unemployed and the economically active population by occupations. The more the jobseekers there are, the less wages in the occupations are reduced. The technical calculation of the wage reduction index in accordance with the number of unemployed in the occupations (formula 314) is identical to the calculation of the wage growth index in accordance with the shortage of employees in occupations (formula 313), so it is not explained again. In total, these indexes form the index of the change of the wage, formula 315:

$$AIK_{\rm Pt} = \begin{cases} DSR \land (APT_{\rm Pt} - ASB_{\rm Pt}), APT_{\rm Pt} - ASB_{\rm Pt} > 0\\ -DSR \lor (APT_{\rm Pt} - ASB_{\rm Pt}), APT_{\rm Pt} - ASB_{\rm Pt} \le 0 \end{cases}$$
(315)

where

AIK<sub>P</sub> - index of the change of wage; DSR - labour demand and supply impact limits; APT<sub>P</sub> - wage growth index in accordance with the shortage of employees in occupations;

ASB<sub>P</sub> - wage reduction index in accordance with a number of unemployed in occupations;

 $N_P$  - number of employed by occupations; P - occupation.

Index of the change of wage is based on the difference between wage growth index (in accordance with the shortage of employees in occupations) and wage reduction index (in accordance with the number of unemployed in occupations). By analysing the behaviour of the model, it is established that theoretically there are circumstances, when at the relatively small number of economically active population in occupation; this wage can be changed too quickly with the negligible change in employment or the number of vacancies. In order to prevent this shortcoming, the model assumes that the labour shortage or surplus cannot change wages more than intended within the bounds of the impact in comparison to the wages, balanced in the long-term period. These bounds are titled in the model as labour demand and supply impact limits. These limits determine the percentage of the change of the wage from the wages, balanced in the long-term period, in accordance with the market conditions, i.e. labour supply and demand. Labour demand and supply impact bounds limit the difference between wage growth and reduction indexes in formula 315. Formula 316 presents the calculation of the wages:

$$A_{\rm Pt} = MA \lor (IA_{\rm Pt} \times (1 + AIK_{\rm Pt})), \tag{316}$$

where  $A_P$  - wages; MA - minimum wage; IA<sub>P</sub> - balanced wages in the long-term period; AIK<sub>P</sub> - index of the change of wage; P - occupation.

As formula 316 shows, the wage calculation is based on the multiplication of the wages, balanced in the long-term period, by the index of the change of the wages. This multiplication cannot be lower than the minimum wage in the country.

$$IA_{\rm P}(t) = IA_{\rm P}(t_0) + \int_{t_0}^{\rm T} (IAP_{\rm P}) \, dt \,, \qquad (317)$$

where

 $IA_P$  - balanced wages in the long-term period;  $IAP_P$  - wage growth in accordance with the productivity growth; P - occupation. The changes of the wages, balanced in a long-term period, are determined by the wage growth in accordance with the productivity growth, formula 318.

$$IAP_{Pt} = \begin{cases} 0, APP_{Pt} - IA_{Pt} < 0\\ APP_{Pt} - IA_{Pt}, APP_{Pt} - IA_{Pt} \ge 0 \end{cases},$$
(318)

where

 $IAP_P$  - wage growth in accordance with the productivity growth;  $APP_P$  - wages by occupations in accordance with the productivity growth;  $IA_P$  - balanced wages in a long-term period; P - occupation.

Wage growth in accordance with the productivity growth is related to the relation algorithm of wage and sector productivity, and reflect the difference of the wage and wages, balanced in a long-term period (under growth conditions), determined in the algorithm. This means that the balanced wages (in a long-term period) in the model are inflexible on the reduction side.

The relation algorithm of wage and sector productivity is presented in Figure 2.62.



Figure 2.62. Algorithm of relation of wage and sector productivity

Algorithm of relation of wage and sector productivity in the base period calculates the average wages in sectors and average wage structure in the sectors by occupations (in order to calculate these indexes, the algorithm calculates inter-indexes "wage fund of the sectors" and "average wages in the sectors by occupations"). Multiplying the average wages in the base period by the productivity growth, the average wages in the sectors are obtained. This assumption is based

on the principle that the wages and productivity growth should be similar in a longer period. On the basis of the average wages in the sectors and the wage structures by occupations in the base period, the average wages in the sectors by occupations are calculated in accordance with productivity growth. The base period structure indicates that, throughout the forecasting period the occupational input structure in the formation of GDP remains unchanged, accordingly, the wage structure remains unchanged as well (e.g. drivers' wages are higher than workers' wages, in a fixed constant ratio). Next, on the basis of the average wages in the sectors by occupations (according to productivity growth), by the means of the inter-index "wage fund of the sectors in accordance with the productivity growth", the wages by occupations are obtained (according to productivity growth). The nature and appliance of this index has already been investigated (formula 318).

The equations of the correlation algorithm of wage and sector productivity are presented in formulas 319 - 326.

$$NAF_{\rm PNt} = N_{\rm PNt} \times A_{\rm Pt} , \qquad (319)$$

where

 $NAF_{PN}$  - wage fund of the sectors;  $N_{PN}$  - number of employed in the occupation by sectors;  $A_P$  - wages; P - occupation; N - sector.

Wage fund of the sectors is calculated by multiplying the wages (by occupations) by the number of employed in the sector (formula 319). The index is calculated for the entire forecasting period, but is only used in the base period ( $t_0$ ).

$$A_{\rm PNt} = \frac{NAF_{\rm PNt}}{1 \vee N_{\rm PNt}},\tag{320}$$

where

 $A_{PN}$  - average wages in the sectors by occupations; NAF<sub>PN</sub> - wage fund of the sectors; N<sub>PN</sub> - number of employed in the occupation by sectors; P - occupation; N - sector.

Taking into account the statistical data shortage, formula 320 reflects the assumption that in all sectors the certain occupation wages do not differ. This means that the wages (vector) determined in formula 316 can be applied in any sector. Formula 320 extends vector up to matrix, taking into account a number of sectors. This index is calculated for the entire forecasting period, but is only used for the base period ( $t_0$ ).

$$A_{Nt_0} = \frac{\sum_{i \in P} NAF_{PNt_0}^{i}}{\sum_{i \in P} N_{PNt_0}^{i}},$$
(321)

where

 $A_{Nt0}$  - average wages in the sectors in the base period;  $NAF_{PN}$  - wage fund of the sectors;

 $N_{PN}$  - number of employed in the occupation by sectors; P - occupation; N - sector.

The average wage in the sector is calculated by dividing the wage fund of the sector by number of employed in the sector. According to this formula, the index is calculated only for the base period  $(t_0)$ .

$$AS_{PNt_0} = \frac{A_{PNt_0}}{A_{Nt_0}}, \qquad (322)$$

where

 $AS_{PNt0}$  - structure of average wage in the sectors by occupations in the base period;  $A_{PN}$  - average wages in the sectors by occupations;  $A_{Nt0}$  - average wages in the sectors in the base period; P - occupation; N - sector.

The average wage structure in the sector by occupations is calculated by dividing the average wage in the sector by occupations by the average wage in the sector. This index reflects how the wage exceeds (or is less than) the average wage level in the sector. The index is calculated only in the base period ( $t_0$ ).

$$A_{\rm Nt} = A_{\rm Nt_0} \times \frac{RK_{\rm PNt}}{RK_{\rm Nt_0}}, \qquad (323)$$

where

A<sub>N</sub> - average wages in the sectors;

 $A_{Nt0}$  - average wages in the sectors in the base period;

RK<sub>N</sub> - productivity index;

 $RK_{\text{Nt0}}$  - labour productivity index in the base period (efficiency index in the base period); N - sector.

Average wages in the sectors grow in direct proportion to labour productivity growth (formula 323).

$$APP_{PNt} = AS_{PNt_0} \times A_{Nt_0}, \qquad (324)$$

where

 $APP_{PN}$  - average wage in the sectors by occupations in accordance with the productivity growth;

AS<sub>PNt0</sub> - structure of average wage in the sectors by occupations in the base period;

A<sub>N</sub> - average wages in the sector;

P - occupation;

N - sector.

Using the initial (base period) average wage structure in the sectors by occupations, the average wages in the sectors are distributed by occupations (324 formula).

$$NAFPP_{\rm PNt} = N_{\rm PNt} \times APP_{\rm PNt}, \qquad (325)$$

where

 $NAFPP_{PN}$  - wage fund of the sectors in accordance with the productivity growth;  $N_{PN}$  - number of employed in the occupation by sectors;

 $\ensuremath{\text{APP}_{\text{PN}}}\xspace$  - average wage in the sectors by the occupations in accordance with the productivity growth;

P - occupation;

N - sector.

Wage fund of the sectors (in accordance with productivity growth) is an inter-index, which assists in calculating the average wage by occupation (in accordance with productivity growth).

$$APP_{\text{Pt}} = \frac{\sum_{i \in N} NAFPP_{\text{PNt}}^{i}}{1 \vee \sum_{i \in N} N_{\text{PNt}}^{i}},$$
(326)

where

 $\begin{array}{l} APP_P \mbox{-} wages by occupations in accordance with the productivity growth; \\ NAFPP_{PN} \mbox{-} wage fund of the sectors in accordance with the productivity growth; \\ N_{PN} \mbox{-} number of employed in the occupation by sectors; \\ P \mbox{-} occupation; \\ N \mbox{-} sector. \end{array}$ 

Wages by occupations (in accordance with productivity growth) are calculated by distributing the occupational wage fund (in accordance with productivity growth) (summarily by sector) by the number of employed.

In the sub-model of labour prices there are indexes, which are not directly involved in the sub-model activity, but they have an important role in model activity - among them there are average wage at the levels of education and occupational comparison matrix. The calculation of the index "average wages at the levels of education" is described in formulas 327 - 329. The calculation of the occupational comparison matrix is presented in formulas 330 - 334.

In order to calculate the index "average wage at the levels of education", the inter-index "number of employed by occupations and levels of education" is calculated in the model, formula 327:

$$PLN_{PLt} = \sum_{i \in D} \sum_{k \in J} KN_{DPJLt}^{ki} , \qquad (327)$$

where

 $PLN_{PL}$  - number of employed by occupations and levels of education;  $KN_{DPJL}$  - total number of employed;

D - gender;

P - occupation;

J - field of education;

L - level of education.

As formula 327 shows, the number of employed by occupations and levels of education is a simple number of employed by gender and fields of education. Using the resulting indexes, the

whole amount of wages of all workers by levels of education and occupations are calculated, formula 328:

$$LPAF_{PI_t} = PLN_{PI_t} \times A_{P_t}, \qquad (328)$$

where

 $\label{eq:linear} \begin{array}{l} LPAF_{PL} \mbox{-} wage fund by levels of education and occupations;} \\ PLN_{PL} \mbox{-} number of employed by occupations and levels of education;} \\ A_{P} \mbox{-} wages; \\ P \mbox{-} occupation; \\ L \mbox{-} level of education. \end{array}$ 

The average wages at the levels of education have been calculated from the wage fund by levels of education and occupations and the number of employed at the levels of education, formula 329:

$$A_{\rm Lt} = \frac{\sum_{i \in P} LPAF_{\rm PLt}^{i}}{1 \vee \sum_{i \in P} PLN_{\rm PLt}^{i}},$$
(329)

where

 $\begin{array}{l} A_L \mbox{ - average wages at the levels of education;} \\ LPAF_{PL} \mbox{ - wage fund by levels of education and occupations;} \\ PLN_{PL} \mbox{ - number of employed by occupations and levels of education;} \\ P \mbox{ - occupation;} \\ L \mbox{ - level of education.} \end{array}$ 

Average wages by the levels of education, the relative difference of the levels is one of the indexes, which makes the population to continue their education.

Wage level affects not only the choice of the level of education, but also the wish of the employed to change occupation. Labour price sub-model determines the privileged occupations along with the other indexes, i.e. occupations with the higher wage level than in other occupations; as well as the opportunities to change occupation are determined, taking into account higher wage level, formulas 330 - 334.

$$A_{\rm PPt} = \sqrt{A_{\rm Pt} \times A_{\rm Pt}^{\ T}} , \qquad (330)$$

where

 $A_{PP}$  - wage vector transformation to matrix;  $A_P$  - wages; P - occupation.

The wage vector (column) in formula 330 is multiplied by the transposed vector (vector - row). The square root has been taken from the multiplication. In the result of the mathematical operations the matrix has been obtained, whose rows represent wages by occupations. This interindex does not have any economic sense, but it provides a mathematically technical implementation of the model. It allows calculating the wage correlation matrix, formula 331:

$$AAM_{\rm PPt} = \frac{A_{\rm PPt}}{1 \lor A_{\rm Pt}} \times MP , \qquad (331)$$

where  $AAM_{PP}$  - wage correlation matrix;  $A_{PP}$  - wage vector transformation to matrix;  $A_{P}$  - wages; MP - permutation matrix; P - occupation.

Wage correlation matrix represents that the wage in a particular occupation is lower in comparison with other occupations. In accordance with the low quality of the available data (wages are not known for certain occupations, or they contain a zero level), 331 formula determines the lowest wage level (1 LVL); it provides the functionality of the model, regardless of the quality of the available data. Permutation matrix function in formula 330 is related to the conversion of the wage correlation matrix to the triangular matrix form, so the condition is fulfilled  $a_{ij} = 0$ , if i>j. Triangular matrix compares the wage of the particular occupation with other wages only once (once for each), it is the economic advantage of this type of matrix. Permutation matrix is an upper triangular matrix ( $a_{ij} = 1$ , if  $i \le j$  and  $a_{ij} = 0$ , if i>j.)

The privileged occupations have been calculated from the wage correlation matrix, formula 332:

$$PAAM_{PPt} = \begin{cases} AAM_{PPt}, AAM_{PPt} < 1\\ 0, AAM_{PPt} \ge 1 \end{cases},$$
(332)

where  $PAAM_{PP}$  - privileged occupations;  $AAM_{PP}$  - wage correlation matrix; P - occupation.

As the wage correlation matrix represents how many times the wage of a particular occupation is lower in comparison with the other occupations, this index in the privileged occupations is less than one, i.e., the wages in the privileged occupations are higher. Indexes of privileged occupations reflect how many times the wage in a particular occupation is more attractive in comparison with other occupations (the smaller this index is, the higher the wage is).

The calculation of privileged occupational matrix is based on the privileged occupations, formula 332:

$$PAAM 2_{PPt} = \begin{cases} 1, PAAM_{PPt} > 0\\ 0, PAAM_{PPt} \le 0 \end{cases},$$
(333)

where  $PAAM2_{PP}$  - matrix of privileged occupations;  $PAAM_{PP}$  - privileged occupations; P - occupation.

Privileged occupational matrix forms the permutation matrix based on the privileged occupations, i.e., indicates that the occupations are economically related, not indicating the degree of the relation.

Privileged occupational matrix is used to calculate matrix of privileged occupations for occupational mobility, formula 334:

$$PPMPM_{PPt} = PAAM 2_{PPt} \times PAM_{PPt}, \qquad (334)$$

where

PPMPM<sub>PP</sub> - matrix of privileged occupations for occupational mobility; PAAM2<sub>PP</sub> - matrix of privileged occupations; PAM<sub>PP</sub> - occupational compliance matrix; P - occupation.

Privileged occupational matrix cannot be used directly. Despite the attractiveness of the occupations, it is not always possible to change one occupation to another; it is determined by the specifics of an occupation. The occupational compliance matrix has previously specified the occupations that can be changed to other occupations. Privileged occupational matrix observes the specifics of these occupations for the occupational mobility (formula 334).

Matrix of privileged occupations for the occupational mobility is the last labour price submodel index.

### **3. MODEL USER GUIDE**

SDM has been developed by Powersim Studio 9 (service release 2) software, the created file is adapted to the earlier versions of the program, starting with Powersim Studio 7 Feature Pack 2 (Service Release 5 (Studio 7)). Model application possibilities are related to Powersim software requirements: Microsoft Windows, as well as Powersim hardware requirements. The developed model (along with Powersim software) is using in the performance process up to 1.90 GB of RAM. Model optimal performance is largely dependent on the Powersim software version and specificity. Powersim software, according to the available versions, determines the model simulation process speed. 30 step simulation (from 2010 to 2040 year, one step per year) takes 5 minutes on the average. Hardware capacity increase over the requirements specified in the Powersim version specification, does not increase the model simulation speed.

The model has been designed for the staff with basic Powersim knowledge. In order to improve the model application possibilities for the staff, the model contains the model graphical user interface. Despite the fact that the model can be used by the staff with basic Powersim knowledge, it is not recommended to change the model connections without any solid experience in Powersim. Application of the model is related to the offered opportunities in the model control interface, preparation of data (MS Excel environment) and start of the simulation. Model automatically ensures the data exchange from the external environment (MS Excel) to the model and vice versa.

# 3.1. Model interface

The model consists of three files: Powersim file with SDM.sip model and two MS Excel files - SDM\_in.xls and SDM\_out.xls. SDM\_in.xls - file that automatically ensures the data exchange from the external environment to the model. The model results are automatically integrated into SDM\_out.xls file.

To start the work with the model, SDM.sip file should be opened. By opening SDM.sip file, the model is displaying on the first page the model control interface, Figure 3.1.



Figure 3.1. Model control interface

Model control interface is designed to control the policy analysis options (education and migration fields) to change the integrated constants in the model in order to obtain the number of calculated vacancies, as well as to ensure additional data export.

Model control interface provides the following policy analysis options: firstly, the model user is being offered to choose the application type of study places: the provided study places in the external environment (from MS Excel) or the modelled study places in the model. Secondly, it is being offered to choose the international migration application type: the provided migration in the external environment (from MS Excel) or the modelled migration in the model. By choosing the modelled migration in the model, it is possible to determine the significance of the impact of the modelled migration on the labour market, i.e., it is possible to change the index of "migrant ratio in labour shortage problem solution" (in Latvian: "imigrācijas īpatsvars darbaspēka trūkuma problēmas risināšanā"). This index reflects the part of the labour shortage covered by the international migration (the other part is being covered by the education system). By determining the index value greater than 1, the situation is being modelled, in which international migration exceeds over labour shortage.

Model control interface allows changing the constants integrated into the model. To start the constant correction, it is necessary to go to the constant correction (using the URL "constant correction" (in Latvian: "konstanšu labošana")), Figure 3.2.

Galvenas un ārpus apakšmodeļu konstantes									
Prognozēšanas horizonts	2041,00	Nosaka prognozēšanas pēdējo gadu (neieskaitot).							
Nodarbinātības izmaiņu laiks	1,00 yr	Atspoguļo laiku, kurā ir iespējams veikt nodarbinātības izmaiņas atbilstoši IKP izmaiņām.							
Profesijas svarīguma kritērijs	0,03	Nosaka nozarei svarīgo profesiju minimālo īpatsvaru no visiem nozarē nodarbinātajiem.							
Nodarbinātības izmaiņu etapu novērtēšanas koeficients	1,00	Atspoguļo, kā mainoties nozaru nodarbinātībai par i (%), iestājas jauns izmaiņu posms, kurā profesiju izmaiņu struktūra ir citāda.							
Darba atrašanas cerību zuduma laiks	1,50	Darba atrašanas cerību zuduma laiks.							
Profesionālās mobilitātes koeficients	1,00	Atspoguļo nodarbināto daļu, kuri mainītu darba vietu, ja būtu pieejams izdevīgāks piedāvājums.							
Priekšrocīga vecuma struktūra [vg15_19]	6,93e-3	Atspoguļo no darba devēju skatījuma darbinieku prekšrocīgo vecumu (pa 5 gadīgām grupām).							
Priekšrocīga vecuma struktūra [vg20_24]	0,10	Priekšrocīga vecuma struktūras daļa vecuma grupai no 20 gadiem līdz 24 gadiem.							
Priekšrocīga vecuma struktūra [vg25_29]	0,13								
Priekšrocīga vecuma struktūra [vg30_34]	0,12								
Priekšrocīga vecuma struktūra [vg35_39]	0,13								
Priekšrocīga vecuma struktūra [vg40_44]	0,12								
Priekšrocīga vecuma struktūra [vg45_49]	0,13								
Priekšrocīga vecuma struktūra [vg50_54]	0,12								
Priekšrocīga vecuma struktūra [vg55_59]	0,09								
Priekšrocīga vecuma struktūra [vg60_64]	0,03								
Priekšrocīga vecuma struktūra [vg65_69]	0,01								
Priekšrocīga vecuma struktūra [vg70_74]	7,63e-3								
Darbaspēka pieprasījuma un piedāvājuma ietekmes robežas	0,03	Atspogujo, cik būtiski darbaspēka trūkums vai pārpalikums var ietekmēt algas (daja).							
Minimālā alga	200,00 Ls	Minimälä alga.							

### Figure 3.2. Fragment of the constant correction tables

The fragment of the second page, table of constants correction, is shown in Figure 3.2. The constants integrated into the model are grouped by sub-models in the separate tables, and the logical relationship of constants is underlined by the colours (e.g., single equation/algorithm constants or structure constants (where the table is successively reflecting each structure element)). The constants integrated into the model are described in subsection 3.3.

Simultaneously with the constant regulation, the model is necessary to be ensured by the data, i.e. to fill SDM\_in.xls file. SDM\_in.xls file data description is available in Annex A.1, but the technical description of the file is available in Annex A.7.

By calculating the designed labour reduction and changes of labour demand in the first forecasting year, the model determines the vacancies, created in the base year. The vacancies in the base year is a specific parameter, which can be provided on the basis of both statistic and model calculations for the base year. If this parameter has been provided on the basis of the statistics, its completion is standard (as all other SDM\_in.xls file tables). If the model calculations are required, it is necessary to perform the following procedure:

1. Model control interface goes to the link "Generated number of initial vacancies" (in Latvian: "Ģenerētais sākotnējais vakanču skaits").

- 2. Open the element "Generated number of initial vacancies" (in Latvian: "Ģenerētais sākotnējais vakanču skaits") (clicking it twice).
- 3. Go to "Value".

rofesijas: <in rows=""></in>	• Iz	glītības_limer In rows>	ni: D:	zimums: (In columns	> • [	zglītības_jo <in column<="" th=""><th>mas: s&gt; 💌</th><th>Unit: Cilvēks/yr</th><th></th></in>	mas: s> 💌	Unit: Cilvēks/yr	
Profesij	Izglītīb	Vīrieši j141	Vīrieši j142	Vīrieši j211	Vīrieši j212	Vīrieši j213	Vīrieši j214	Vīrieši j215	•
p0	1DLI	0,00	0,00	0,00	0,00	0,00	0,00	0,00	
p0	2AI	0,00	0,00	0,00	0,00	0,00	0,00	0,00	
p0	3Plpai	0,00	0,00	0,00	0,00	0,00	0,00	0,00	
p0	4VvAPvi	0,00	0,00	0,00	9,97	0,00	0,00	0,00	
p0	5VpApZp	0,00	0,00	0,00	0,00	0,00	0,00	0,00	
p111	1DLI	0,00	0,00	0,00	0,00	0,00	0,00	0,00	
p111	2AI	4,17	0,00	0,00	0,00	0,00	0,00	0,00	
p111	3Plpai	0,00	0,00	0,00	0,00	0,00	0,00	0,00	
p111	4VvAPvi	0,00	0,00	0,00	0,00	0,00	0,00	0,00	
		•						F	
Import E	rom File	Import	From Cir	hoard (		Export		<b>,</b>	

- 4. Press "Export…".
- 5. Export data to clipboard (without the data subject).

•

6. Put data from clipboard to the designed place of SDM\_in.xls file (the "Paste" function in Excel) (file technical description is available in Annex A.7).

7. In the end of the procedure to save SDM\_in.xls file.

The manual transmission of calculated data from Powersim to Excel ensures the option in base year to fill the vacancies formed in the base year.

To run the model, Powersim software standard toolbar "Simulation" button is being used, Figure 3.3. To run the model, only one button "Toggle Play" ( $\blacktriangleright$ ) should be used. More details on the toolbar "Simulation" button are presented in Powersim software documentation.

Simulation	
M 🕨 🗶 📲 🗕 🕅 🤡 🔂 🔛	- 😒 -

Figure 3.3. Standard toolbar "Simulation" buttons

Upon completion of the simulation, the model automatically transfers the modelling results to SDM\_out.xls file.

In order to export the result data from the model, i.e. data that is not automatically integrated into SDM\_out.xls file, the model control interface involves the function "extended data export" (in Latvian: "paplašinātā datu eksportēšana"). The application of this function allows quickly and easily find the key model parameters and technical elements, which can be the basis for the creation of extended data export. In addition, the result data acquisition is associated with the technical operations in Powersim environment, so that this option is designed for advanced Powersim users and has not been described in detail. In the option "extended data export" the available data list is presented in subsection 3.4.

Powersim SDM.sip file contains the model and its relationships. Model relationship correction is not recommended without a thorough Powersim knowledge. All SDM.sip file pages that are not reflected in the model control interface, are technical pages, but they are still important and contain the model equations. Table 3.1 shows the layout of the model sub-models by model pages.
No.	Name of the page	Brief description				
1.	Home	Reflects the model control interface; home, first page of the model.				
	(Sakums)					
2.	Tables	Reflects the model control interface; tables of constant corrections.				
	(Tabulas)					
3.	Start_vac	Reflects the model control interface; number of generated initial				
	(Sak_vak)	vacancies.				
4.	Output	Reflects the model control interface; copies of the significant				
	(Izvadei)	parameters and technical elements, which are provided for the				
		formation of the additional output.				
5.	Sectors_dem	Reflects the sub-model for the formation of the labour demand by				
	(Nozares_piep)	sectors				
6.	Occupation_dem	Reflects the sub-model for the formation of the labour demand by				
	(Profesijas_piep)	sectors and occupations.				
7.	Education_dem	Reflects the sub-model for the formation of the labour demand by				
	(Izglītības_piep)	occupations and education.				
8.	Education_dem _ind	Reflects the sub-model for the formation of the labour demand by				
	(Izglītības_piep_Noz)	sectors, occupations and education.				
9.	Gender_dem	Reflects the sub-model for the formation of the labour demand by				
	(Dzimums_piep)	occupations and gender.				
10.	Education_supp	Reflects the education attainment sub-model.				
	(Izglītība_pied)					
11.	Lifelong	Reflects the lifelong education attainment sub-model.				
	educationl_supp					
	(Mūžizgl_pied)					
12.	Population_supp	Consists of demography and employable population analysis sub-				
	(Iedzīvotāji_pied)	models.				
13.	Ec_activity_supp	Reflects the population economic activity sub-model.				
	(Ek_aktivitāte_pied)					
14.	Migration_supp	Reflects the population international migration sub-model.				
	(Migrācija_pied)					
15.	Market	Reflects the labour market sub-model.				
	(Tirgus)					
16.	Sectors_employm	Reflects the sub-model for employed population division by				
	(Nozaru_nodarb)	occupations and sectors.				
17.	Wage	Reflects the wage definition sub-model.				
	(Alga)					

Table 3.1. SDM.sip file description

## **3.2.** Data input and model update

SDM\_in.xls - file that automatically ensures the data exchange from the external environment to the model. By updating SDM\_in.xls file, the model can be used over a longer period of time. The file has multiple pages that contain different types of data. More detailed relative analysis of the file pages and data is presented in Annex A.1.

SDM\_in.xls file pages, the included technical format and technical data requirements are described in detail in Annex A.7.

## **3.3. Integrated constants in the model**

In order to make the model function adequately, it is necessary to set the forecasting start and end time in the modelling settings (Simulation/Simulation settings in the Powersim computer program).

Calendar:	limestep:	LITTLE C
Rank w	1.00 yr	2010.01.01
Start time: 2010.01.01.	Stop time: 2040.01.01.	Reset Simulation
Simulation speed		Automatic history
Real time		Save state
Custom	ms/time step	Animate reference data

## Figure 3.4. Powersim software program "Simulation/Simulation settings" window

Along with the "Simulation/Simulation settings", it is important to restore the element "Forecasting horizon" (in Latvian: "Prognozēšanas horizonts") (which is explained in Annex A.6). Upon determination of the values for the constants, it is not required to change them.

In model pages constants are marked as Powersim constants, and additionally are filled in yellow colour (apart from "Forecasting horizon", which is filled in red). The integrated constants into model are described in Annex A.6.

Mostly the integrated constants into the model are to be determined by means of survey of the experts. The majority of the constants does not change in the short and medium time period (periods less than 10 years).

## **3.4. Model output tables**

The model results are automatically integrated into SDM\_out.xls file. The list of SDM\_out.xls file pages and tables is presented in Figure 3.2.

No.	Name of the page	List of tables / description	
1.	Employment	Employment demand structure by sectors (people)	
	(Nodarbinātība)		
2.	OccupationsD	Employment demand structure by occupations (people);	
	(ProfesijasD)	Replacing demand structure by occupations (people);	
		Expansion demand structure by occupations (people)	
3.	Occupations	Population structure by occupations (people)	
	(ProfesijasS)		
4.	OccupationsA	Active population structure by occupations (people)	
	(ProfesijasA)		
5.	OccupationsN	Employment structure by occupations (people)	
	(ProfesijasN)		
6.	PG_LS_LN	Technical page, can be excluded form file	
7.	Ratio of occupations	Employment demand division by sectors and occupations (structure	
	(Profesiju_īpatsv)	of sectors-occupations) (%)	
8.	EducationD	Employment demand structure by levels, fields of education (people)	
	(IzglītībaD)		

## Table 3.2. **SDM\_out.xls fail description**

9.	EducationS (IzglītībaS)	Population structure by levels, fields of education (people)				
10.	EducationA	Active population structure by levels, fields of education (people)				
	(IzglītībaA)					
11.	EducationN	Employment structure by levels, fields of education (people)				
	(IzglītībaN)					
12.	Ratio of occupations	Employment demand structure by fields, levels of education and				
	(Izglītības_īpatsv)	occupations (people)				
13.	Demography	Population by age for (one-year) groups and gender (people);				
	(Demogrāfija)	Total mortality of the population by gender (people);				
		Total birth rate of the population by gender (people);				
		Total emigration of the population by gender (people);				
		Total immigration of the population by gender (people)				
14.	Level of education	Population by levels of education (people);				
	(lzgl_lim)	Number of school students, pupils and students by levels of				
		education (people);				
		Admission of school students, pupils and students by levels of				
		education (people);				
1.5	ן ח	Number of graduates by levels of education (people)				
15.	Pop_age_gend	Population by age (5-year) groups and gender (people)				
16	(ledz_sk_vec_Dz)	Active nonvelotion by ease (5 year) groups and conder (nearla)				
10.	(Iedz_sk_Vec_Dz A)	Active population by age (3-year) groups and gender (people)				
17.	Pop_age_gend N	Employed population by age (5-year) groups and gender (people)				
	(Iedz_sk_Vec_Dz N)	(people)				
18.	Sectors (Nozares)	Employed population by sectors (people)				
19.	Ind-prof	Employed population by sectors and occupations (people)				
	(Noz-prof)					
20.	WagesP	Average wage in occupation (LVL)				
	(AlgasP)					
21.	WagesN	Average wage in the sector (LVL);				
	(AlgasN)	Average wage (LVL).				
22.	VakanciesD	Free vacancies by gender.				
	(VakancesD)					
23.	VakanciesP	Free vacancies by occupations.				
	(VakancesP)					
24.	VakanciesIz	Free vacancies by occupations, fields and levels of education.				
	(VakancesIz)					

To simplify access to the reflected results in the table 3.2 and other model results, the copies of the significant parameters and technical elements have been created in the model control interface - "Output" page, provided for obtainment of the additional results. Description of the "Output" page is presented in table 3.3.

Tuble 5.5. Description of the page "Output							
No.	Name of the element	Dimensions	Measure				
1.	Employment demand by occupations and sectors	Occupations,	People				
		sectors					
2.	Occupational structure of employment demand by sectors	Occupations,	Part				
		sectors					

Table 3.3. Description of the page "Output"

3.	Employment demand by occupations, fields, levels of education and gender	Levels of education, fields of education, occupations, gender	People
4.	Occupational structure of employment demand by fields and levels of education	Levels of education, fields of education, occupations	Part
5.	Employment demand by occupations, fields, levels of education and sectors AJ	Levels of education, fields of education, occupations, sectors AJ	People
6.	Employment demand by occupations, fields, levels of education and sectors KR	Levels of education, fields of education, occupations, sectors KR	People
7.	Admission of students by fields and levels of education	Levels of education, fields of education	People per year
8.	Number of graduates by fields and levels of education	Levels of education, fields of education	People per year
9.	Number of students by fields and levels of education	Levels of education, fields of education	People
10.	Number of optimized study places by fields and levels of education	Levels of education, fields of education	People per year
11.	Population	Age, gender	People
12.	Birth rate by gender	Gender	People per year
13.	Mortality	Age, gender	People per year
14.	Emigration by age groups and gender	Age, gender	People per year
15.	Immigration by age groups and gender	Age, gender	People per year
16.	Emigration	Agegroups,gender,occupations,fieldsofeducation,economicactivity groups	People per year

17.	Immigration	Age groups, gender, occupations, fields of education, economic activity groups	People per year
18.	Population by levels, fields of education occupations and economic activity groups	Age groups, gender, occupations, fields of education, economic activity groups	People
19.	Population by age groups, gender and economic activity groups	Age groups, gender, economic activity groups	People
20.	Total number of vacancies	Gender, occupations, fields of education, levels of education	People
21.	Total number of employed	Gender, occupations, fields of education, levels of education	People
22.	Number of employed by age groups and gender	Age groups, gender	People
23.	Total number of jobseekers	Gender, occupations, fields of education, levels of education	People
24.	Replacing demand	Gender, occupations, fields of education, levels of education	People per year
25.	Number of employed in the occupation by sectors	Occupations, sectors	People
26.	Salaries by occupations	Occupations	LVL
27.	Average salaries in the sector	Sectors	LVL
28.	Average wage	-	LVL

# 4. DESCRIPTION OF THE RESULTS OBTAINED WITH THE MODEL, INTERPRETATION AND RELIABILITY OF THE RESULTS

Model result evaluation is based on the experiment (simulation) series result analysis. Series of experiments includes several simulations based on the model, labour market and the national economy real statistical data, by changing the exogenous forecasting data in the experiments, as well as switching on and off the provided functions in the model. Table 4.1 visually describes the experiments.

No	GDP	Study places	Migration policy	Name of
	forecast			experiment
1.		Study places without changes in accordance with the base year	The closed system has been modelled, no migration, it is not affecting the labour market	GDP without changes, scenario without impact
2.	GDP without changes	Study places has beenmodelledbyoptimizingthenationalpolicyeducationthe	The closed system has been modelled, no migration, it is not affecting the labour market	GDP without changes, optimization scenario of the study places
3.		Study places without changes in accordance with the base year	Emigration and immigration has been defined in the model in accordance with the market demand	GDP without changes, migration impact scenario
4.	GDP is	Study places without changes in accordance with the base year	The closed system has been modelled, no migration, it is not affecting the labour market	GDP growth impact, scenario without impact
5.	regularly in all sectors throughout the whole	Study places has beenmodelledbyoptimizingthenationalpolicyeducationthe	The closed system has been modelled, no migration, it is not affecting the labour market	GDP growth impact, optimization scenario of the study places
6.	period, 3% per year	Study places without changes in accordance with the base year	Emigration and immigration has been defined in the model in accordance with the market demand	GDP growth impact, migration impact scenario
7.	GDP is	Study places without changes in accordance with the base year	The closed system has been modelled, no migration, it is not affecting the labour market	GDP decrease impact, scenario without impact
8.	regularly in all sectors throughout the whole	Study places has beenmodelledbyoptimizingthenationalpolicyeducationin	The closed system has been modelled, no migration, it is not affecting the labour market	GDP decrease impact, optimization scenario of the study places
9.	period, 3% per year	Study places without changes in accordance with the base year	Emigration and immigration has been defined in the model in accordance with the market demand	GDP decrease impact, migration impact scenario

1 able 4.1. Description of experiment
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The table 4.1 shows that the experiments have been grouped into 3 groups, depending on the GDP forecasting conditions, and the scenarios without impact and the changes of study places and migration impacts are analysed in each group.

Results of the experiments are presented in table 4.2. They are analysed in more detail after the table.



#### Table 4.2. Modelled scenario analysis table



	20,0	20,0	25,0
	■ Without impact ■ Study places ■ Migration	■ Without impact ■ Study places ■ Migration	■ Without impact ■ Study places ■ Migration
	15,0	15,0	15.0
<b>Mortality</b> , people per thous.	10,0	10,0	10,0
	5,0	5,0	5,0
	0,0	0,0	0,0
	0,0	0,0	0,0
	2011 2020 2030 2040 -5,0	2011 2020 2030 2040	-5,0 2011 2020 2030 2040
Natural increase, thous,	-10,0	10.0	-10,0
people	-15,0	-10,0	-20,0
	-20,0	-15,0	-25,0
	-25,0 Without impact Study places Migration	-20,0 Without impact Study places Migration	-30,0 Without impact Study places Migration
	60,0 ■Emigration ■Immigration ■Migration ratio	60,0 Emigration Migration Migration ratio	40,0
	40,0	40,0	20,0
Migration,	20,0	20,0	0,0
thous. people	0,0	0,0	2011 2020 2030 2040
	-20,0 2011 2020 2030 2040	-20,0	-20,0
	-40,0	-40,0	-40,0







The first parameter analysed in the model is the **labour demand**. Labour demand is analysed separately for each scenario. Each scenario evaluates the expansion and replacing labour demand, comparing to the initial (base period) labour demand.

Expansion labour demand indicates the changes of the labour demand (expansion) in case of change (growth) of GDP. Model results indicate the connection between the changes of the labour and GDP: the growing GDP causes the growth of labour demand; decline in GDP causes the decline in labour demand; in case of constant GDP, the labour demand remains unchanged.

By summing up the initial (base) labour demand and expansion labour demand, the total labour demand is being obtained, which dynamics depends on GDP development forecasts.

The replacing labour demand indicates the labour demand, which is being formed due to the need to replace the employed population, who was die, as well as population, leaving their working places due to the retirement age. Replacing demand is significant, taking into account that more than 55% of the Latvian employed is older than 40 years and are going to leave the labour market in the coming 30 years (forecast period). The replacing labour demand for labour is mainly related to the population and employed initial (base) age structure, and this parameter does not differ significantly by the scenarios.

The analysed table is visually showing that before 2040 the replacing labour demand exceeds the half of the initial (base) labour demand, which means that most of the current specialists in the market should meet the deputies.

The next analysed parameter in the model is the **population**. Population in all developed scenarios in the country has declined.

By analysing the change of population, it is determined that GDP growth is almost not affecting the population (excluding migration scenario), but the decline in GDP significantly accelerates the decline in population (compared to scenarios without changes of GDP).

Migration policy impact on the population is different. Over the first ten years, following the initial large negative balance of migration, migration always reduces the population. The following situation is not so simple: depending on the changes of GDP, the labour demand changes as well, and then the population follows. With the growth of GDP, the labour demand also grows, migration covers missing labour demand, the population in the country grows as well, which is evident in the GDP growth scenario from 2030 year. Without changes of GDP, in the scenario the shortage of labour due to decline in population will appear only in 2040, and migration is to cover it. In case of decline in GDP there is no labour shortage, while there is a labour surplus, which is being reduced by migration. In this case, there is a decline in population.

By summarizing the above mentioned, it can be concluded that the model reflects the fact that in case of GDP growth the migration increases the population, but in case of decline in GDP the migration reduces the population.

Part of the population is the **population of working age**. These two parameters are mutually correlating, so the dynamics will not be re-examined.

The **birth rate** is significantly dependent on the population and structures. In the coming decade the birth rate decline by 15% is expected. This is related to the fact that during the same period, the population and the number of women of reproductive age will decrease by the same amount. It is important to note that GDP growth is mostly not affecting the changes of the birth rate, but the decline in GDP accelerates the decline in birth rate.

The growth of the birth rate is predicted till the 2030, which is related to the age structure of the population: by assessing the population by 5-year old groups, now the 5-year-old to 9-year-old population exceeds 10-year-old to 14-year-old population, but 0-year-old to 4-year-old population exceeds 5-year-old to 9-year-old population. This will ensure the growth of the population and the number of women of reproductive age from 2020 to 2030th, which will accordingly increase the birth rate. However, in case of decline of GDP, the present growth of birth fails to overcome another influencing factor - GDP reduction, and in these scenarios the growth of birth rate is not expected.

Significant impact on the birth rate has the migration processes. Model indicates that migration always increases the birth rate. It reflects that the emigration structure depends on the current structure of the population, but immigrants are mostly of reproductive age and are able to raise the birth rate.

The next parameter analysed in the model is **mortality**. Mortality significantly depends on the population and structure. Despite this fact, the growth of GDP reduces mortality, but the decrease in GDP increases mortality. Besides the impact and optimization of study places the scenarios provide a sustained increase of mortality. However, migration reduces mortality. This is explained by the fact that the emigration structure is based on the current structure of population, but immigrants are generally relatively young people, who reduce the mortality rate.

Birth rate and mortality form the natural growth rate, which is analysed below. **Natural growth** is negative in all scenarios. Natural growth is dependent on changes of GDP - with the growth of GDP, the natural growth increases; with the decrease of GDP, the natural growth decreases. Considering that migration increases birth rate and reduces mortality, it is evident that migration significantly increases the natural growth.

By analysing the **migration** in the model, instead of nine simulations, only three are being investigated, which were included in the migration modelling and these three simulations are analysing three parameters: emigration, immigration and balance of migration.

In the analysed scenarios emigration initially significantly exceeds immigration. Emigration amounts are not changing significantly in all scenarios. Its gradual reduction is being explained by the gradual decline in population.

According to the included relations, immigration covers the labour shortage, which explains the rapid increase in immigration until 2020th. In subsequent years, immigration is dependent on changes of GDP: with the growth of GDP, immigration increases, with the decrease of GDP, immigration decreases. This is related to the changes of labour demand and arising of labour shortage.

Balance of migration reflects the difference between immigration and emigration.

**The economically active population** is related to the working age population. These two parameters are mutually correlating; the model assumes that the economically active population is directly proportional to the population by 5-year-old groups and gender according to externally provided indexes (for the provided participation level). This explains why a whole economically active population is proportional to the population of working age.

However, taking into account the changes of age structure, it leads to the overall participation level fluctuations. **Participation level** will increase slightly by 2020th, by 2030 it will decrease by 2 percentage points, and by 2040 will increase (in scenarios without impact and optimization of study places). The population age structure in immigration scenarios is different: the population is younger. In the younger groups the participation level is higher. With the increase of population in younger groups, the overall participation level increases as well.

**Number of employed persons** is a complex index, which affects both the economically active population, from the perspective of the supply, and the labour demand from the perspective of the demand. To simplify the interpretation of the parameter, it is analysed along with the next parameter - **employment rate**.

Dynamics of the number of employed persons and employment is different in all scenarios.

In addition to the changes of GDP, the major impact on the employment in the scenario group has the decline in the number of employees, as indicated by the decline in the number of employed along with the preservation of the employment rate. The scenario does not provide the growth of GDP, and, accordingly, the increase of living standard and the rapid demand expansion in occupations. In such circumstances occurs the short-term satisfaction of the occupational demand, education attainment, but then it remains non-demanded. In order to slightly increase the number of employed and employment rate in this scenario, the significant investments in the education system (through continuous re-qualification) are required. In the present results the employment in the scenario of optimization of study places is slightly lower than in the scenario without impact. This

is related to the fact that the model optimizes the study places in accordance with the population, but in the scenario without impact 1.35 places will be financed per one applicant by 2040th. Model reflects the by increasing the number of study places, employment may be increased. And to increase the number of employed, the number of study places should never be reduced. The migration policy in the same scenario group can solve the labour problem - increase both employment and employment rate.

The decline in the number of employed in the GDP growth scenario group is slower, at the same time there is a growth of employment rate. This indicates the fact that the main role in employment in this scenario group belongs to the growth of GDP and the resultant increase of labour demand. In case of the growth of labour demand, the incentive to attain education also increases, and in this scenario group the optimization of study places increases both the number of employed and the employment rate (the number of study places, comparing to the base year, is being significantly reduced). However, the education system is not able to completely solve all problems of labour market due to lack of population. But migration policy can solve labour problems, by increasing both employment and employment rate.

The decline in the number of employed and decrease of employment rate in GDP reduction scenario group is being significantly affected by the decline in GDP and the resultant decline in labour demand. None of the influencing factors can increase the number of employed. But the migration policy may result in faster decrease of the number of employed; there is also decline in the total population, which leads to slower decrease of the employment rate.

The next analysed parameter is the **number of jobseekers**. The number of jobseekers is a complicated parameter, which is being affected by the economically active population and the number of employed. To simplify the explanation of the parameter, it is being analysed along with the **unemployment rate**.

Dynamics of the number of jobseekers and unemployment is different in all scenarios.

In the scenario group without changes of GDP the number of jobseekers and the unemployment rate is declining in accordance with the decline in population (over the first forecasting years the unemployment rate remains stable high, considering that the new study places are not created, and the replacing demand is minimal). This scenario group shows the fiasco of education system, in the case of absence of GDP growth - occurs the short-term satisfaction of the occupational demand, education attainment, but then it remains non-demanded. Therefore, the optimization of study places (mainly the decline in accordance with the population) causes the increase of the number of jobseekers and unemployment rate. Migration policy in this scenario group reduces both the number of jobseekers and the unemployment rate.

In the GDP growth scenario group, the number of jobseekers and the unemployment rate are decreasing, taking into account the decline in the number of population and increase of the number of employed (due to the growth of GDP and increase of labour demand). This scenario group represents that the education system optimization is acting over the medium term. This is related to the objective reasons: in order to prepare specialists with higher education, at least 4 years are necessary, additionally the time is required from the formation of the shortage of occupations to the problem identification and opening of new study places, etc. Education policy in this scenario group allows reducing the number of employed and the unemployment rate prior to other affecting elements. However, its effectiveness is being also reduced, with the unemployment rate reaching 2-3%. In this scenario group migration reduces the speed of decline in the number of jobseekers. This is related to the fact that migration increases the population. Migration policy options are also limited, when the unemployment rate is on 2-3% level.

The number of jobseekers and the unemployment rate in GDP reduction scenario group are steadily increasing. This indicates the fact that in case of decline in GDP, neither the education, nor the immigration policy can reduce unemployment. However, the migration policy reduces the population, which is not creating so rapid increase of the number of jobseekers and unemployment rate, as in the other scenarios. The next analysed parameter is the **average wage**. Model provides long-term increase of the wage along with the increase of the labour productivity, the steady increase of the wage is evident in GDP growth scenario group. This parameter is also affected by other factors, such as labour supply and demand ratio, occupational structure. In total, these factors lead to a small increase of the wage in other scenario groups. In migration scenarios, compared to other scenarios, the wage increase is slightly faster. It is based on the changes of the occupational: the shortage of specialists is being formed in country, which is being filled by immigrants. Increase of the number of qualified specialists changes the labour market structure, by increasing the number of highly paid specialists, raising the average wage in the country.

# 5. DESCRIPTION OF THE TIME CURVE OF THE DETAILED FORECASTING PROCESS

SDM model uses a number of statistical data types prepared by CSB as the input data. This determines the necessity for the regular data update. Table 5.1 presents data update time schedule. Table 5.1 presents data update and updating time by months. Data update detailed information (for each parameter with an accuracy to the day) is presented in Annex A.9.

Month	I	П	Î	IV	V	VI	VII	VIII	IX	X	XI	XII
Data group .	-				•		, 11	,				
Demography data (population)												
Demography data (birth rate, mortality)												
Education data (school)												
Education data (other)												
Labour data												
GDP data												
Wage data												
Forecasting data of the MoE												

 Table 5.1. Data update time schedule

SDM model operates with the annual data. Accordingly, the annual data is being prepared by CSB once a year. The only data that can be updated on a quarterly basis in order to increase the quality of forecasts is data on wage.

Along with the CSB statistical data update, it is necessary to revise the forecasts, i.e. assumption data. Among them are the GDP forecast, target labour sector-occupational structure, forecast of the economically active part of the population. Depending on the simulation target, the update of the parameter "number of the new state-financed study places" may be needed.

With the global change of national economy, labour market or education system may need to update the following elements: period of the studies, occupation compliance matrix, time indexes of sectorial labour productivity. It is not necessary to annually update these elements.

SDM model data update and forecasting time consumption is presented in table 5.2. Table 5.2 visually reflects the activity groups and the required time. More detailed information of data update and forecasting time is presented in Annex A.10.

No.	Activity group	Total time of the group
1.	Data update	13 hours
2.	Assumption data review	2 hours
3.	Special data review (option)	10 hours
4.	Implementation of the procedure before modelling	0,2 hours
5.	Modelling process (one simulation)	0,2 hours
6.	Full calibration of the model (option)	48 hours
7.	Examination of results	0,2 hours

 Table 5.2. Data update and forecasting time consumption

Table 5.2 shows that the data update, development of one scenario, modelling and obtainment of the results require about 2 days. If the changes of global national economy, labour market and education system have been observed, the model adjustment can take longer time. This is largely related to the special data review and model calibration. The developed model contains updated data, which is the latest available at the moment of the model transfer; the model contains a full calibration. Review of the optional opportunity is not required in the next 3-year period.

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Annex A.

## The annexes to the body text of the technical documentation

Annex A.1.

# Data specification and sources used in modelling

Data	SDM_in file	Included data							
No.	sheet name	Data name	Data source	Dimensions	Classification	Absolute / relative	Measure	Brief description	
1.	Employment / Nodarbinātība	Employment by sectors in the base year	CSB OVERVIEW OF WAGE STRUCTURE (5-work); OVERVIEW OF WORK (3-work)	Sectors	At NACE 2 red. branch (letter) level	Absolute	people	Reflects the employment by sectors in the base year.	
2.	GDP / IKP	GDP forecast GDP in the base year	MoE CSB overview table IK04. GROSS DOMESTIC PRODUCT BY TYPE OF ACTIVITY	Sectors	At NACE 2 red. branch (letter) level	Absolute	Actual prices, thous. lats	ReflectsGDPforecastsbysectors.ReflectsGDPbysectors inthebase year. </td	
4.	Indexes / Koeficienti	Time indexofchangesoflabourproductivityproductivityofthe sectorsof	Experts	Sectors	At NACE 2 red. branch (letter) level	Absolute	-	Reflects the time indexes of changes of labour productivity of the sectors.	
5.	Occupations / Profesijas	Divisionofemployedbysectorsandoccupationsinthe base year	CSB OVERVIEW OF WAGE STRUCTURE (5-work); OVERVIEW OF WORK (3-work)	Sectors, occupations	Classifier of the occupations of the Republic of Latvia, occupations at the	Absolute	people	Reflects the division of employed by sectors and occupations in the base year.	
6.		Target labour sectorial- occupational structure	MoE		3-character level; At NACE 2 red. branch (letter) level	Relative	part	Reflects the target labour sectorial-occupational structure	
7.	Education / Izglītība	Division of employed by levels of	CSB OVERVIEW OF WAGE STRUCTURE (5-work); OVERVIEW	Level of education, Field of	First four levels of the Latvian education	Absolute	people	Reflects division of employed by levels, fields of education occupations.	

		education, fields, occupations	OF WORK (3-work)	education, occupations	classification code; Classifier of the occupations of			
8.		Occupational compliance matrix	MoE		the Republic of Latvia, occupations at the 3-character level		-	Reflects the correspondence of occupations to the education (occupational compliance matrix)
9.	Demography / Demogrāfija	Population by age and gender in the base year	CSB overview table IS06. AGE STRUCTURE OF MEN AND WOMEN AT THE BEGINNING OF THE YEAR	One-year age groups, gender	-	Absolute	people	Reflects the population by age and gender in the base year.
10.		Population mortality by age and gender	CSB overview table IM01. DEATHS BY AGE AND SEX		-		people	Reflects the population mortality by age and gender.
11.		Alive born people by age of the mother	CSB overview table ID02. NUMBER OF BORN BY GENDER and ID03. ALIVE BORN PEOPLE BY AGE OF MOTHER	CSB data in Excel file by 5-year-old groups are automatically regularly divided by one-year-old age groups.	-		people	Reflects the number of alive born people by age of the mother.
12.	Dem_Dr	Population by	CSB OVERVIEW OF	5-year-old age	First four levels of	Absolute	people	Reflects the population by levels
13.	Dem_Ai	levels of education in the	WAGE STRUCTURE	groups,	the Latvian			of education in the base year. Each sheet reflects only one level
14.	Dem_Plpai	base year	OF WORK (3-work)	of education,	classification			of education (Dr - doctor's
15.	Dem_VvAPvi			occupation,	code; Classifier of			degree; Ai - academic education
16.	Dem_vpApZp			activity	the occupations of the Republic of Latvia, occupations at the 3-character level			and second level vocational education; Plpai - first level higher vocational education; VvAPvi - general secondary, vocational and vocational secondary education; VpApZp - basic general education, vocational educations with basic education and primary education).
17.	Primary_pupils / Sak_skoln	Number of pupils in the	CSB overview table IZG09. NUMBER OF	Study year (class), gender	-	Absolute	people	Reflects the number of pupils by classes and gender in the base

		base year	EDUCATED IN GENERAL ACADEMIC SCHOOLS BY CLASSES (at the beginning of the school year)					year.
18.	Students / Studējošie	Mastering of vocational education with the basic education	OVERVIEWS PROF-2 and PROF-2m OF VOCATIONAL EDUCATIONAL INSTITUTIONS	Study year, field of education, gender	First three levels of the Latvian education classification code.	Absolute	people	Reflects the number of students in the base year. Each table reflects only one study level.
19.		Masteringofvocationalandvocationalsecondaryeducationsecondary						
20.		Mastering of first level higher vocational education						
21.		Earning of bachelor's degree						
22. 23.		Earningofmaster's degreeEarningofdoctor's degree						
24.	Admission / Uzņemšana	Mastering of vocational education and vocational secondary education	OVERVIEWS PROF-2 and PROF-2m OF VOCATIONAL EDUCATIONAL INSTITUTIONS	Fields of education, gender	First three levels of the Latvian education classification code.	Absolute	people	Reflects the number of enrolled pupils and students in the base year. Each table reflects only one study level.
25.		First level higher vocational education						
26.		Bachelor's degree						

27.		Master's degree						
28.		Doctor's degree						
29.	Education_supply / Izglītības_pied	Duration of the studies	MoE	Level of education, Fields of education	First four levels of the Latvian education classification code.	Absolute	years	Reflects the duration of the studies by fields and levels of education.
30.	Additional matrix / Papildmatrica	Additional matrix	-	Occupations	Classifier of the occupations of the Republic of Latvia, occupations at the 3-character level	Absolute	-	The sheet has the technical support function. The sheet contains the permutation matrix (upper triangular matrix, $a_{ij} = 1$ , if $i \le j$ and $a_{ij} = 0$ , if $i > j$ .). The sheet is not provided for the changes made by user.
31.	Salaries / Algas	Monthly salaries in the base year.	CSB OVERVIEW OF WAGE STRUCTURE (5-work); OVERVIEW OF WORK (3-work)	Occupations	Classifier of the occupations, occupations at the 3-character level	Absolute	LVL	Reflects the monthly salaries by occupations in the base year.
32.	Employ_D / Nodarb_Dr	Employed population in	CSB OVERVIEW OF WAGE STRUCTURE	5-year-old groups; fields	First three levels of the Latvian	Absolute	people	Reflects the employed population in the base year. Each table
33.	Employ_A / Nodarb_Ai	the base year	(5-work); OVERVIEW OF WORK (3-work)	of education; occupations;	education classification			reflects only one study level in accordance with the above
34.	Employ_Plpai / Nodarb_Plpai			gender	code; Classifier of the occupations of			reflected cancellations.
35.	Employ_VvAPvi / Nodarb VvAPvi				the Republic of Latvia,			
36.	Employ_ VpApZp / Nodarb_VpApZp				occupations at the 3-character level			
37.	Active_V_str / Aktīvo_V_str	Part of economically active population	MoE	5-year-old groups; occupations; gender	-	Relative	part	Reflects the part of economically active population (of the total population of working age).
38.	Study_places / Stud_viet	Number of new state-financed study places	МоЕ	Level of education, Fields of education	First three levels of the Latvian education classification code;	Absolute	Number of places	Reflects the number of study places by levels and fields of education.
39.	Initial_vac / Sakot_vak	Number of vacancies in the base year	Generated in SDM.sip file; data can be obtained in another way	Occupations; Level of education;	First three levels of the Latvian education	Absolute	Number of vacancies	Reflects the number of vacancies in the base year.

			(statistically or from experts).	fields of education; gender	classification code; Classifier of the occupations of the Republic of Latvia, occupations at the 3-character level			
40.	Educ_sector / Izgl_noz	Division of employed by levels of education, fields, occupations and sectors in the base year.	CSB OVERVIEW OF WAGE STRUCTURE (5-work); OVERVIEW OF WORK (3-work)	Sectors; Level of education; fields of education; occupations	Sector at the section (leer) level; First three levels of the Latvian education classification code; Classifier of the occupations of the Republic of Latvia, occupations at the 3-character level	Absolute	people	Reflects the division of employed by levels of education, fields, occupations and sectors in the base year.

Annex A.2.

Period of changes of productivity by sectors	7 milex 71.2.
Sectors (according to the grouping of MoE)	Index
Agriculture, Forestry and Fishing (A)	0,78
Mining and quarrying (B); Electricity, gas, steam and air conditioning supply	2,40
(D); Water supply, sewerage, waste management and remediation activities (E)	
Manufacturing (C)	0,96
Construction (F)	2,67
Wholesale and retail trade; repair of motor vehicles and motorcycles (G)	1,485
Transportation and storage (H)	1,47
Accommodation and food service activities (I)	3,00
Information and communication (J)	2,50
Financial and insurance activities (K)	2,30
Real estate activities (L)	2,80
Professional, scientific and technical activities (M); Administrative and support	5,00
service activities (N); Other service activities (S);	
Activities of households as employers; undifferentiated goods- and services-	
producing activities of households for own use (T)	
Public administration and defence; compulsory social security (O); Activities of	8,00
extra territorial organizations and bodies (U)	
Education (P)	0,74
Human health and social work activities (Q)	0,79
Arts, entertainment and recreation (R)	1,205

Annex A.3.

# Compliance of the significance of occupational groups (3-character level) to the criteria by sectors in 2010

	Number	
	occupatio	Total
	nal	ratio in
Sectors	groups	the sector
Agriculture, hunting and forestry (A)	8	75%
Fishing (B)	6	91%
Mining and quarrying (C)	10	90%
Manufacturing (D)	7	42%
Electricity, gas and water supply (E)	7	67%
Construction (F)	12	81%
Wholesale and retail trade; repair of motor vehicles, motorcycles and		
personal and household goods (G)	8	69%
Hotels and restaurants (H)	4	81%
Transport, storage and communication (I)	8	52%
Financial intermediation (J)	10	86%
Real estate, renting and business activities (K)	10	62%
Public administration and defence, compulsory social security (L)	10	70%
Education (M)	7	69%
Health and social care (N)	8	80%
Other utility, social and personal services (O)	7	55%

Annex A.4.

	Dropout index values									
	Basic general education, vocational education with basic education	General secondary vocational education and vocational secondary education	Vocational education	First level higher vocational education	Academic education and second level vocational education	Doctor's degree				
Dropout indexes	0%	13,4%	19,4%	17,2%	4,0%	19,9%				

Annex A.5.

Privileged age structures index values

Age group	Privileged age structure	Privileged age structure of the release from work
vg15_19	0,7%	30,6%
vg20_24	10,0%	2,1%
vg25_29	13,3%	1,6%
vg30_34	12,1%	1,8%
vg35_39	12,6%	1,7%
vg40_44	12,0%	1,8%
vg45_49	13,0%	1,6%
vg50_54	12,4%	1,7%
vg55_59	9,0%	2,3%
vg60_64	3,2%	6,6%
vg65_69	1,0%	20,4%
vg70_74	0,8%	27,8%
Total	100%	100%

Annex A.6.

No	Name	Sub-model	Value	Measu	Recommended	Description	Role in the model
110	1 vunic	Sub mouer	value	re	constraints	Description	Note in the model
1.	Time of change of employment	Sector demand	1	year	1	Reflects the time, when it is possible to make employment changes in accordance with GDP changes.	Ensures the changes of the labour demand in accordance with GDP changes. The time defines the extent of synchronization of GDP and changes of labour demand (at index 1 - changes are synchronized).
2.	Forecasting horizon	Occupational demand	2041	year	Forecasting last year (excluding)	Determines the forecasting last year.	Ensures uniform changes of labour occupational structure from the beginning to the end of the forecasting.
3.	Measure of importance of occupations	Occupational demand	0,03	part	0,03 (determined statistically)	Determines the minimum ratio of the occupation from all, which is important to sector.	Occupations, which ratio in the sector exceeds their value, are considered to be the sector primary occupations; the labour demand in these occupations varies in proportion to GDP
4.	Estimation index of the stages of the change of employment	Occupational demand	1	%	1	Reflects the new stage of changes coming along with the change of employment in the sector by $i$ (%), which has the different structure of changes of occupation.	Under conditions of long-term growth the demand is being increasing not only for primary occupations, but also for other occupations. The index determines the growth limits, which cause the demand changes for "supplementary occupations".
5.	General secondary education dropout	Education supply	0	part	determined statistically	Dropout index in the basic general education.	Determines the dropout part of the students.
	ratio						
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6.	General secondary vocational education dropout index	Education supply	0,134	part	determined statistically	Dropout index.	
7.	First level higher vocational education dropout index	Education supply	0,194	part	determined statistically	Dropout index.	
8.	Bachelor's degree dropout index	Education supply	0,172	part	determined statistically	Dropout index.	
9.	Master's degree dropout index	Education supply	0,04	part	determined statistically	Dropout index.	
10.	Doctor's degree dropout index	Education supply	0,2	part	determined statistically	Dropout index.	
11.	Women dropout index	Education supply	0,378	part	determined statistically	Dropout index.	
12.	Bachelor's index	Education supply	0,715	part	determined statistically	Reflects the bachelor's degree ratio in the vacant study places in the academic and second level vocational education.	Divides the academic and second level occupation in bachelor's and master's degrees (study places).
13.	Longstanding balanced education continuation index from VP AP	Education supply	0,98	part	determined statistically	Reflects longstanding education continuation ratio after GBE (general basic education) into VE (vocational education).	Determines the initial part of the graduates, which continues the education at the higher level. (Index is being used only in the base period, afterwards it is being recalculated).
14.	Time of change of education continuation index	Education supply	2	years	1≤x≤7	Time of change of education continuation index.	Determines the time of change of education continuation index

	VP AP						
15.	Longstanding balanced education continuation index from VP AP VV	Education supply	0,619	part	determined statistically	Reflects longstanding education continuation ratio.	Determines the initial part of the graduates, which continues the education at the higher level. (Index is being used only in the base period, afterwards it is being recalculated).
16.	Time of change of education continuation index VP AP VV	Education supply	2	part	1≤x≤7	Time of change of education continuation index.	Determines the time of change of education continuation index
17.	Longstanding balanced education continuation index from VP AP Arod	Education supply	0,35	part	determined statistically	Reflects longstanding education continuation ratio.	Determines the initial part of the graduates, which continues the education at the higher level. (Index is being used only in the base period, afterwards it is being recalculated).
18.	Time of change of education continuation index VP Arod	Education supply	2	years	1≤x≤7	Time of change of education continuation index.	Determines the time of change of education continuation index
19.	Longstanding balanced education continuation index from PLPaI BLI	Education supply	0,53	part	determined statistically	Reflects longstanding education continuation ratio.	Determines the initial part of the graduates, which continues the education at the higher level. (Index is being used only in the base period, afterwards it is being recalculated).
20.	Time of change of education continuation index PLPaI BLI	Education supply	2	years	1≤x≤7	Time of change of education continuation index.	Determines the time of change of education continuation index
21.	Longstanding balanced education continuation index	Education supply	0,1	part	determined statistically	Reflects longstanding education continuation ratio.	Determines the initial part of the graduates, which continues the education at the higher

	from AI DLI						level. (Index is being used only in the base period, afterwards it is being recalculated).
22.	Time of change of education continuation index AI DLI	Education supply	2	years	1≤x≤7	Time of change of education continuation index.	Determines the time of change of education continuation index
23.	Initial education attainment ratio BLI by PLPaI	Education supply	0,0039 6	part	determined statistically	Reflects the number of students earning bachelor's degree in the base year, if the previous education was PLPaI.	Determines the initial part of the graduates, which continues the education at the higher level. (Index is being used only in the base period, afterwards it is being recalculated).
24.	Lifelong education (LE) Longstanding balanced education continuation index from VP AP	Lifelong education	0,098	part	determined statistically	Reflects longstanding education continuation ratio in the lifelong education.	Determines the initial part of the graduates, which continues the education at the higher level. (Index is being used only in the base period, afterwards it is being recalculated).
25.	LE Time of change of education continuation index VP AP	Lifelong education	2	years	1≤x≤7	Time of change of education continuation index in the lifelong education.	Determines the time of change of education continuation index
26.	LE Longstanding balanced education continuation index from VP VV	Lifelong education	0,0619	part	determined statistically	Reflects longstanding education continuation ratio in the lifelong education.	Determines the initial part of the graduates, which continues the education at the higher level. (Index is being used only in the base period, afterwards it is being recalculated).
27.	LE Time of change of education continuation index VP VV	Lifelong education	2	years	1≤x≤7	Time of change of education continuation index in the lifelong education.	Determines the time of change of education continuation index

28.	LE Longstanding balanced education continuation index from VP Arod	Lifelong education	0,0035	part	determined statistically	Reflects longstanding education continuation ratio in the lifelong education.	Determines the initial part of the graduates, which continues the education at the higher level. (Index is being used only in the base period, afterwards it is being recalculated).
29.	LE Time of change of education continuation index VP Arod	Lifelong education	2	years	1≤x≤7	Time of change of education continuation index in the lifelong education.	Determines the time of change of education continuation index
30.	LE Longstanding balanced education continuation index from PLPaI BLI	Lifelong education	0,0106	part	determined statistically	Reflects longstanding education continuation ratio in the lifelong education.	Determines the initial part of the graduates, which continues the education at the higher level. (Index is being used only in the base period, afterwards it is being recalculated).
31.	LE Time of change of education continuation index PLPaI BLI	Lifelong education	2	years	1≤x≤7	Time of change of education continuation index in the lifelong education.	Determines the time of change of education continuation index
32.	Birth rate ratio of boys	Population supply	0,511	part	determined statistically	Birth rate ratio of boys.	Reflects the birth rate ratio of boys
33.	Independent part of birth rate index	Population supply	0,0396	part	determined statistically	Provided to reflect the birth rate changes along with GDP changes.	Reflects that the birth rate depends not only on GDP, but also on other extrasystemic factors. Determines the total impact of extrasystemic factors on birth rate index.
34.	Birth rate index	Population	5,21e-6	part	determined	Provided to reflect the birth rate changes along with GDP	Reflects that birth rate changes of
		suppry			staustically	changes.	GDP.
35.	Maximum total birth rate index per	Population supply	0,0535	part	determined	The maximum birth rate index limit is reflected in the model	Reflects the number of children being born per ann. Per one

	woman				statistically	at the average EU level.	woman of reproductive age.
36.	Birth rate matching time	Population supply	10	years	1≤x≤15	Determines, how fast the birth rate can change in the longer period.	Determines the time of change of birth rate in the longer period.
37.	Mortality reaction to GDP changes	Population supply	1	-	Not defined	Provided to reflect the mortality changes along with GDP changes.	By multiplying the growth rate of total GDP per capita by this index, the changes of mortality index are being calculated, taking into account the GDP growth.
38.	Correction of longstanding balanced mortality indexes	Population supply	0,9	part	0≤x≤1	Provided to reflect the global prolongation of life.	Reflects that the mortality index balanced in the longer period is X from the mortality index in the base year.
39.	Definition of longstanding unoccupied working places	Migration supply	1,5	years	1≤x≤3	Definition of longstanding unoccupied working places.	Defines the time, after which the vacancy is being considered as the longstanding vacancy. The longstanding vacancies promote immigration.
40.	Time to take migration decision	Migration supply	3	years	1 <u>≤</u> x≤5	Time to take migration decision.	The index reduces the emigration (or slows it in the longstanding unfavourable economic conditions). Emigration in the given period is X times smaller than the initial conditions, taking into account that emigration decision is not being taken immediately.
41.	Unemployed emigration propensity	Migration supply	0,05	part	0≤x≤0,1	Unemployed emigration propensity.	Reflects the part of the unemployed and economically inactive population, which can

							potentially emigrate under certain circumstances
42.	Employed emigration propensity	Migration supply	0,01	part	0≤x≤0,05	Employed emigration propensity.	Reflects the part of the employed, which can potentially emigrate under certain circumstances.
43.	Readiness to call foreign workers	Migration supply	0		0≤x≤1	Readiness to call foreign workers.	Reflects the privileges of the "local" employees in the labour market. At the index value 1 the "local" employees does not have benefits, at the index value "0" the "foreign" workers do not get jobs, economic migration does not take place.
44.	Time to get integrated in the Latvian society	Migration supply	1	years	1≤x≤2	Time to get integrated in the Latvian society.	Time to meet labour market legislation formal requirements in order to get a job.
45.	Immigrant age structure	Migration supply	{0,05; 0,4; 0,3; 0,15; 0,05; 0,05; 0; 0; 0; 0; 0; 0; 0}	part	-	Age structure of immigrants reflects that most of immigrants are 25 - 40 years old, i.e. active working age.	The assumption reflects that immigration flow is mostly related to the economically active population migration.
46.	Time of coverage of vacancies for migration	Migration supply	2	years	1≤x≤10	Time, after which the current vacancies cause immigration.	If the longstanding vacancy occurs, it may have random character. This index determines the time, after which the immigration takes place if the vacancy is still relevant
47.	Immigration ratio	Migration	Change	part	0≤x	Labour shortage may be	Political leadership toolbar

	in labour shortage problem solution	supply	s by scenari os			covered by education system or immigration. The ratio reflects the labour shortage problem, which can be resolved by the means of the immigration	
48.	Target wage ratio index	Migration supply	0,87	-	0≤x≤1	Wage ratio index (in Latvia and largest EU countries), which does not provide economic emigration.By d index Norw are with the being	letermining the size of this x, the salaries in Sweden - way and Sweden - Finland being compared. Along the "Wage ratio index" economic migration is g determined.
49.	Wage ratio index 5VpApZp	Migration supply	0,4362	-	0≤x≤1	Wage ratio index (in Latvia and largest EU countries) at the level of education VpApZp.Wage education ratio emig	e ratio index by levels of cation. If the index is ller than the "Target wage index", the economic gration is taking place.
50.	Wage ratio index 4VvAPvI	Migration supply	0,4362	-	0≤x≤1	Wage ratio index (in Latvia and largest EU countries) at the level of education.	
51.	Wage ratio index 3PLPaI	Migration supply	0,4362	-	0≤x≤1	Wage ratio index (in Latvia and largest EU countries) at the level of education.	
52.	Wage ratio index 2AI	Migration supply	0,4362	-	0≤x≤1	Wage ratio index (in Latvia and largest EU countries) at the level of education.	
53.	Wage ratio index 1DLI	Migration supply	0,4362	-	0≤x≤1	Wage ratio index (in Latvia and largest EU countries) at the level of education.	
54.	Privileged age structure	Market	{0,007; 0,1; 0,133; 0,121; 0,126;	part	Definition based on th number of employed b age groups.	sReflectstheprivilegedageIndexestructureoftheemployeesthatffromtheperspectiveoftheyemployers(by5-year-oldofagroups).fromfromfrom	xes reflect the probability the vacancy may be ipied by a certain employee certain age group (ranging n 15-year-old group to 74-

			0,12; 0,13; 0,124; 0,091; 0,032; 0,010; 0,008}				year-old group, by 5-year-old groups). (Total probability equals to 1). Is applied to the first stage of employment. (Secondly, if a vacancy is not occupied, the age structure is not being observed.)
55.	Occupational mobility index	Market	1	part	0≤x≤1	Reflects the part of employed, which will change their working places, if the more beneficial offers are available.	An index value 1 indicates that all employees will change their working places as soon as the better place is available.
56.	Labour demand and supply impact limits	Wage	0,03	part	0≤x≤0,5	Reflects the extent of the impact of the labour shortage or surplus on the salaries.	Reflects that the balanced salaries in the longer period (justified by productivity) can range in short-term periods within the limits of the index. Taking into account the labour demand and supply.
57.	Minimum wage	Wage	200	LVL	In accordance with the legislation	Minimum wage.	Minimum wage.
58.	Supply and demand geographical correlation index	Market	1	-	0≤x≤1	In the several diagrams are repeating: the compliance of the jobseekers, the compliance of the jobseekers to 4VvAPvI, the compliance of the jobseekers to 3PLPaI, the compliance of the jobseekers to 2AI, the compliance of the jobseekers to 1DLI. Reflects the demand and supply geographical compliance coefficient.	Reflect the supply and demand geographical correlation index. Level 1 indicates that supply and demand geographically coincide.

### Technical description of SDM\_IN.XLS file.

**Employment** (nodarbinātība) sheet format:

The sheet contains a table that reflects employment by sectors in the base year (people), the table starts in A1 cell, the data has been entered, starting from B3 cell. The table has a vertical orientation. A column is designed to reflect the names of the sectors. B column should include the data. B2 cell sums up the number of employed by sectors. Figure 2.1 reflects the visual appearance of the table (Print Screen).



#### Figure 2.1. Employment sheet data format

**GDP** (IKP) sheet format:

The sheet contains two tables: GDP forecasts by sectors and GDP by sectors in the base year (in current prices, thous. LVL).

The table "GDP forecasts by sectors" starts in A1 cell, the data has been entered, starting from B3 cell. Sectors in the table are reflected vertically, but GDP forecasts by years - horizontally. A column is designed to reflect the names of the sectors, starting from B column the data should be entered. First row represents forecasting year; second row sums up the number of employed by sectors. Figure 2.2 reflects the visual appearance of the table (Print Screen).

	A	В	С	D	E	F	G	Н	
1		2012	2013	2014	2015	2016	2017	2018	
2	IKP Prognoze	12679679	12679679	12679679	12679679	12679679	12679679	12679679	1
3	(A) Lauksaimniecība, mežsaimniecība un zivsaimniecība	568 974	568 974	568 974	568 974	568 974	568 974	568 974	
4	BDE	648 450	648 450	648 450	648 450	648 450	648 450	648 450	
5	(C) Apstrādes rūpniecība	1 793 613	1 793 613	1 793 613	1 793 613	1 793 613	1 793 613	1 793 613	1
6	(F) Büvniecība	776 119	776 119	776 119	776 119	776 119	776 119	776 119	
7	(G) Vairumtirdzniecība un mazumtirdzniecība; automobiļu un motociklu rer	2 141 221	2 141 221	2 141 221	2 141 221	2 141 221	2 141 221	2 141 221	2
8	(H) Transports un uzglabāšana	1 653 998	1 653 998	1 653 998	1 653 998	1 653 998	1 653 998	1 653 998	1
9	(I) Izmitināšana un ēdināšanas pakalpojumi	215 043	215 043	215 043	215 043	215 043	215 043	215 043	
10	(J) Informācijas un komunikācijas pakalpojumi	477 557	477 557	477 557	477 557	477 557	477 557	477 557	_
11	(K) Finanšu un apdrošināšanas darbības	480 199	480 199	480 199	480 199	480 199	480 199	480 199	1
12	(L) Operācijas ar nekustamo īpašumu	968 009	968 009	968 009	968 009	968 009	968 009	968 009	_
13	MNST	987 968	987 968	987 968	987 968	987 968	987 968	987 968	1
14	ou	874 850	874 850	874 850	874 850	874 850	874 850	874 850	_
15	(P) Izglītība	532 120	532 120	532 120	532 120	532 120	532 120	532 120	1
16	(Q) Veselība un sociālā aprūpe	345 188	345 188	345 188	345 188	345 188	345 188	345 188	_
17	(R) Māksla, izklaide un atpūta	216 370	216 370	216 370	216 370	216 370	216 370	216 370	
18									_
19									
20	IKP bāzes gadā	2011							
21	IKG041. IEKŠZEMES KOPPRODUKTS PA DARBĪBAS VEIDIEM (NACE 2	12679679							
22	(A) Lauksaimniecība, mežsaimniecība un zivsaimniecība	568 974							
23	BDE	648 450							
24	(C) Apstrādes rūpniecība	1 793 613							
25	(F) Büvnledba	776 119							
26	(G) Vairumtirdzniecība un mazumtirdzniecība; automobiļu un motociklu rer	2 141 221							
27	(H) Transports un uzglabāšana	1 653 998							
28	(I) Izmitināšana un ēdināšanas pakalpojumi	215 043							
29	(J) Informācijas un komunikācijas pakalpojumi	477 557							
30	(K) Finanšu un apdrošināšanas darbības	480 199							
31	(L) Operācijas ar nekustamo īpašumu	968 009							
32	MNST	987 968							
33	ou	874 850							
34	(P) izglītība	532 120							
35	(Q) Veselība un sociālā aprūpe	345 188							
36	(R) Māksla, izklaide un atpūta	216 370							
	▶ Nodarbinātība IKP Koeficienti Profesijas / Tzr	ultina / De	mogrāfia	Dem Dr	Dem Ai	Dem Plazi	Dem VvA	Dvi / Dem	V

Figure 2.2. GDP sheet data format

**Index** (koeficienti) sheet format:

The sheet contains a table that reflects the time indexes of the changes of sectorial labour productivity, the table starts in A1 cell, the data has been entered, starting from B2 cell. The table has a vertical orientation. A column is designed to reflect the names of the sectors. B column should contain the data. Figure 3.2 reflects the visual appearance of the table (Print Screen).

	А	В	С
1		Laika koef	
2	(A) Lauksaimniecība, mežsaimniecība un zivsaimniecība	0,780	
3	BDE	2,400	
4	(C) Apstrādes rūpniecība	0,960	
5	(F) Büvniecība	2,670	
6	(G) Vairumtirdzniecība un mazumtirdzniecība; automobiļu un motocik	1,485	
7	(H) Transports un uzglabāšana	1,470	
8	(I) Izmitināšana un ēdināšanas pakalpojumi	3,000	
9	(J) Informācijas un komunikācijas pakalpojumi	2,500	
10	(K) Finanšu un apdrošināšanas darbības	2,300	
11	(L) Operācijas ar nekustamo īpašumu	2,800	
12	MNST	5,000	
13	00	8,000	
14	(P) izgitiba	0,740	
15	(Q) Veselība un sociālā aprūpe	0,790	
16	(R) Māksla, izklaide un atpūta	1,205	
17			
18			

Figure 2.3. Index sheet data format

Occupation (profesijas) sheet format:

The sheet contains two tables: division of employed by sectors and occupations in the base year (people) and the target labour sectorial-occupational structure (%).

The table of division of employed by sectors and occupations in the base year (people) starts in A1 cell, the data has been entered, starting from B2 cell. The occupations in the table are reflected vertically, but the sectors - horizontally. A column is designed to reflect the names of the occupations, starting from B column the data should be entered. First row reflects the names of the sectors.

The format of the table "Target labour sectorial-occupational structure (%)" is identical to the format of the table "division of employed by sectors and occupations in the base year" (people). Table "Target labour sectorial-occupational structure (%)" starts in Y1 cell, the data has been entered, starting from Z2 cells. Figure 2.4 reflects the visual appearance of the tables (Print Screen).

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1	Profesije		606	c		6		1	1	ĸ	1 L	MNST	00	,	9								Vilamias	(A) Leukae ()	<li>a) loguvol</li>	(C) Apstel (D	dicking (s	5 Odona 🚯	r) 50 mie (5)	Veinum (h)	Transp (1)	Ismitin 0	() informal)	X) TimerS ()	(L) Operate (	M) Profes (H	() Admire	(D) Velsta p	pirvelde un	a data
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3	£111	172		275		199	83				299	720	1 770			144							p111	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,02	0,01	0,03	0,00	0,00	0,01		
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2	(12)	\$14	253	1 573	1 2 2 2	3 754	1 122	120	482	200	847	2 272	4 675	622	813	512							0121	0.01	0.01	0.01	0.02	0.03	0.02	0.01	0.01	0.05	0.06	0.04	0.05	0.01	0.01	0.00		
	1122			\$20	63	200			220	275		77	112	172									0122	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.00		
7	1121	2.427		115									170										0121	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		
	1122	100	670	3 360	4 275	712	4 210			100		450	205			03							0122	0.00	0.03	0.02	0.00	0.01	0.06	0.00	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.01		
	1122	_				_		_	267		_		24	77									0133	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00		
10	1124	90				719				609			1409	4 479	691	222							0124	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.04	0.00	0.00	0.02	0.06	0.01	0.01		
33	1161	_						1 060			_			49									0141	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		
32	1162		29	27		6 269						164		-									0162	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		
12	1142	_		41		1 227	66		112			640	296	270		946							0142	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.06		
24	1011	_		244	111	282				45		1 050	25	227	24								0211	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.01	0.00	0.00		
15	1010	_						_	43	225	_				61								0212	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00		
18	1010	847										126	112										0213	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		
37	1014	434	236	4 102	1 996	403	284	_			369	1 282	627	34	_	162							0214	0.01	0.02	0.07	0.06	0.01	0.01	0.00	0.00	0.00	0.05	0.03	0.02	0.00	0.00	0.02		
18	1944	110	443	244	82	184	318	163	682	309	208	874	244	112		43							0215	0.00	0.04	0.01	0.00	0.00	0.01	0.01	0.03	0.03	0.03	0.02	0.01	0.00	0.00	0.01		
12	1216			281		24	54		160			1389	344		_								0216	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.03	0.01	0.00	0.00	0.00		
25	1011	_										761	403		7.746								0221	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.17	0.00		
21	1010	_						_			_			100	2.547								6222	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00		
22	1016	_													74								0223	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		
25	114	_				_		-					152		2.062								0224	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00		
24	4946	63										424											6225	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00		
25	1214			101		1.592		-				200	278	262	1.455								6226	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00		
24	4751	_												3 785									6231	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.00		
100	4755	_	_		_	_	_	_					_	1 271	41								0232	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00	0.00		
100	1000	-	_		_	_		-	-	-	-	_	_											1 100		-														

**Figure 2.4. Occupation sheet data format** 

**Education** (izglītība) sheet format:

The sheet contains two tables: "Division of employed by education levels, fields, occupations (people)" and "Occupational compliance matrix".

The division of employed by education levels, fields, occupations starts in A1 cell, the data has been entered, starting from D3 cell.

Occupational compliance matrix begins in A1001 cell, the data has been entered, starting from D1003 cell.

Education sheet data format and visual appearance (Print Screen) are presented in Figure 2.5.

	А	В	С	D		
1				Title of occupation		Title of occupation P
2				Occupation code (in growing sequence, starting from a smaller)		Occupation code P
3	Education level (starting from doctoral level education to the lowest education)	Education field code (in growing sequence, starting with a smaler)	Title of education field		-	
			 Title of education		Dat	a fields
	Education level L	Education field code J	field J			



Figure 2.5. Education sheet data format

Education sheet format is unified for two sheet tables.

#### **Demography** (demogrāfija) sheet format:

The page contains the following tables: "Population by age and gender (people). "Mortality of the population (people) by age and gender", "Alive born people by age of the mother (people)", (by alive born people by age of the mother by 5 year-old groups); as well as the tables providing choice of the index of division of born people by gender (which are not related to the model).

The table of the population by age and gender (people) starts in A1 cell, the data has been entered, starting from B3 cell. The one-year age groups are presented in the tables vertically, but the gender of the population - horizontally. A column is designed to reflect the name of the one-year-old group, starting from B column the data should be entered. Second row reflects the names of the gender.

The format of the table "Mortality of the population by age and gender (people) is identical to the format of the table "Population by age and gender (people). Table "Mortality by age and gender (population)" starts in E1 cell, the data has been entered, starting from F3 cell.

The table of alive born people by age of the mother (people) starts from M5 cell, the data has been entered, starting from N7 cell. The table has a horizontal orientation. 6 row (N6) is designed to reflect the name of the one-year-old group, 7 row (from N7 cell) the data has been entered. The table fields are being filled automatically from the related table "Alive born people by age of the mother by 5 year-old groups". Related table "Alive born people by age of the mother by 5 year-old groups" starts in M1 cell, the data has been entered, starting from N3 cell. These tables have identical format.

Figure 2.6 reflects the visual appearance of the table (Print Screen).

1	Α	В	С	D	E	F	G	н	1	J	К	L	М	N	0	Р	Q	R	S	Т	U	٧v	УX
1	ledzīvotā	ju skaits		2011 gads	Mirstība								Dzīvi dzim	nušie pēc i	nātes ve	cuma (5	.g.g.)						
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4		l 109	04 10540	)	1	. 10	8																
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10		7 102	19 9550	5	7	' 3	4																
11	1	96	36 9232	2	8	5	0																
12	1	93	03 9072	2	9	5	1																

Figure 2.6. Demography sheet data format

#### Dem\_Dr, Dem\_Ai, Dem\_Plpai, Dem\_VvAPvi, Dem\_VpApZp sheet's format:

All sheets have a unified format. Each sheet is reflecting one level of education according to its shortened name. (Dr. - PhD, Ai - academic education and second-level vocational education; Plpai - first level vocational higher education; VvAPvi - secondary general, vocational and vocational secondary education; VpApZp - basic general education, vocational education with the basic education and primary education).

Each sheet contains a table that reflects the population by age (5-year-old), groups, gender, fields of education, occupations, economic activity (people) in the base year. Table starts in A4 cell, the data has been entered, starting from D6 cell.

"Dem" group sheet data format and visual appearance (Print Screen) are presented in Figure 2.7.





Figure 2.7. "Dem" group sheet data format

**Grade** (Sak\_skoln) sheet format:

The sheet contains a single table, which reflects the number of students by classes and gender (people). The table starts in A1 cell, the data has been entered, starting from B2 cell. The learning classes in the table are presented vertically, but the gender of the students - horizontally. The table has a vertical orientation. A column is designed to reflect the names of the classes, B and C columns should contain data. The following table fields are filled automatically from the transposed table (i.e., table, in which the rows and columns are interchanged). Transposed table starts in G1 cell, the data has been entered, starting from H2 cell. Model does not take the data from the transposed table.

The model of the table, presented in the sheet, is divided into two parts: primary and secondary school (table is sequentially reflecting the number of students by classes). Primary and secondary education in the model are analysed at different levels of education.

Figure 8.2 reflects the visual appearance of the table (Print Screen).

	Α	В	С	D	E	F	G	н	1	J	
1		Vīrieši	Sievietes			2010/11	IZG09. IZG	1.klase	2.klase	3.klase	4.k
2	1.klase	10604	9669	Pamatsko	la		zçni	10604	9849	9604	
3	2.klase	9849	9561				meitenes	9669	9561	9230	
4	3.klase	9604	9230								
5	4.klase	9889	9410								
6	5.klase	9409	8840								
7	6.klase	9121	8482								
8	7.klase	9693	8740								
9	8.klase	10036	9063								
10	9.klase	11562	10450								
11	10.klase	5807	7327	Vidusskol	а						
12	11.klase	6338	8118								
13	12.klase	6738	8767								
14											
15											

Figure 2.8. Primary school sheet data format

Student (studējošie) sheet format:

The sheet contains the following tables: "The number of students mastering vocational education with the basic education", "The number of students mastering vocational education with the vocational secondary education", "The number of students mastering the first-level vocational higher education", "The number of students earning Bachelor's degree", "The number of students earning Master's degree", "The number of students earning Doctor's degree".

The sheet table reflects the number of students in the base year by study years, gender, field of education and level (people). Each table presents only one level of education. The table format and visual appearance (Print Screen) are presented in Figure 2.9.



18	arodizglītī	110		120		121		130	
19	Apmacība	Vīrieši	Sievietes	Vīrieši	Sievietes	Vīrieši	Sievietes	Vīrieši	Sievi
20	1	680,5878	1133,412	112,6247	258,3753	2	6	7,116667	17,8
21	2	597,9179	1017,082	92,76233	234,2377	2	6	7,983333	20,0
22	3	499,8217	823,1783	75,85824	169,1418	2	6	3,833333	10,1
23	4	330,5447	662,4553	73,89693	167,1031	0	0	6,25	1
24	Grand Tot	2108,872	3636,128	355,1422	828,8578	6	18	25,18333	63,8
25									
26									
27									
28	pirmā līm	110		120		121		130	
29	Apmacība	Vīrieši	Sievietes	Vīrieši	Sievietes	Vīrieši	Sievietes	Vīrieši	Sievi
30	1	247,1335	361,8665	0	0				
31	2	23,86645	69,13355	6	16				

#### 2.9. Student sheet data format

The tables in the sheet are unified, use the same format. Tables start in different locations, it is reflected by Table 2.1.

Name of the table	Beginning of	First coll of
	Deginning of	
	the table (cell)	data entry
Number of vocational education with the basic education	A3	B5
mastering students		
Number of vocational education and the vocational secondary	A18	B20
education mastering students		
Number of first level vocational higher education mastering	A28	B30
students		
Number of students earning the Bachelor's degree	A39	B41
Number of students earning the Master's degree	A51	B53
Number of students earning the Doctor's degree	A63	B65

Table 2.1. Arrangement of the table of "Student" sheet

By preparing the data in the student sheets, it is important to note that study year may not exceed the duration of the studies (see Education\_supply description of the page).

#### Admission (uzņemšana) sheet format:

The page contains the following tables: "Number of enrolled in vocational education and vocational secondary education", "Number of enrolled in first level vocational higher education", "Number of enrolled for Bachelor's degree", "Number of enrolled for Master's degree", "Number of enrolled for Doctor's degree".

The table reflects the enrolled students and the number of students in the base year by gender, fields and levels of education (people). Each table reflects only one level of education.

Figure 2.10 reflects the visual appearance of the table (Print Screen).

1	Α	В	С	D	E	F	G	Н	1	J	К
1	arodizglītī	bas un pro	fesionālās	vidējās izg	lītības apg	pirmā līm	eņa profes	ionālā aug	stākā izglīt	ība	bakalau
2	izgl_jom	Vīrieši	Sievietes			izgl_jom	Vīrieši	Sievietes			izgl_jon
3	110	729	1103			110	254	365			1
4	120	109	262			120	3	19			13
5	121	3	5			121					13
6	130	6	17			130					1
7	140	114	297			140	7	32			10
8	141	21	120			141	10	258			10
9	142					142	29	29			10
10	150	21	279			150	10	105			12
11	160	45	16			160	7	25			10
12	170	2	19			170	0	0			1
13	210	512	24			210	22	0			2
14	211					211	12	18			2
15	212					212	6	36			2

Figure 2.10. Admission sheet data format

Table has the following format: vertically, in ascending sequence, starting from the lowest, is reflected the code of the field of education, but horizontally - gender (first - men, second - women). Accordingly the data has been entered. The tables in the sheet are unified, use the same format. Tables start in different locations, it is reflected by Table 2.2.

Name of the table	Beginning of	First cell of
	the table (cell)	data entry
Number of enrolled in vocational education and vocational	A1	B3
secondary education		
Number of enrolled in first level vocational higher education	F1	G3
Number of enrolled for Bachelor's degree	K1	L3
Number of enrolled for Master's degree	P1	Q3
Number of enrolled for Doctor's degree	U1	V3

Table 2.2. Arrangement of the table of "Admission" sheet

Education\_supply (izlgītības\_pied) sheet format:

The page contains a table that reflects the duration of the studies by fields and levels of education (years), the table starts in A1 cell, the data has been entered, starting from D2 cell. The table has a vertical orientation. A column is designed to reflect the name of the level of education (from Doctor's degree to lower-level studies), B column - the code of the field of education (in ascending sequence, starting from the lowest), C column - name of the education, D column should contain the data.

The model of the table, presented in the sheet, is divided into five parts in accordance with the levels of education (table successively reflects the duration of the studies). The levels of education in the model are analysed separately.

Figure 2.11 reflects the visual appearance of the table (Print Screen).

	А	В	С	D	E	F
1	lzglītības pakāpe	lzglītības jomas kods	lzglītības jomas nosaukums	Mācību ilgums gadi		
2		1141	Pedagogu izglīt	5,0		
3		i142	Izglītības zinātr	5,0		
4		i211	Vizuāliplastiskā	5,0		
5		i212	Mūzika un skat	5,0		
6		i213	Audiovizuālā m	5,0		
7		i214	Dizains	5,0		
8		i215	Amatniecība	5,0		
9		i216	Lietiškāmāksla	5.0		

Figure 2.11. Education supply sheet data format

The model requires the duration of the studies as the whole numbers. The maximum duration of the studies at each level of education is limited in the following way: basic education, vocational education with basic education and primary education - 9 years, general secondary, vocational and vocational secondary education - 4 years, first level vocational higher education, academic education and second level vocational education, Doctor's degree - 6 years (for each level).

Wage (algas) sheet format:

The page contains one table, which reflects the monthly income by occupations (LVL) in the base year, as well as technical information for the calculation of these salaries. The table starts in D1 cell, the data has been entered, starting from E2 cell. The table has a vertical orientation. D column is designed to reflect the name of the occupational code (in ascending sequence, starting from the lowest), E column should contain the data.

Taking into account that the data on the wage income would be available only in annual terms, as well as in detail at the two-digit level, in order to fill the required table for the table the data contains the algorithm that transforms the annual wage income (two-digit level in occupations) up to monthly wage income at the three-digit level. In order to use this algorithm in the additional table, which starts in A1 cell, the data should be entered (starting from B2 cell).

Figure 2.12 reflects the visual appearance of the table (Print Screen).

	Α	В	С	D	E	F	0
1	Profesijas kods	Gada alga	menesī	Profesijas	Alga		
2	1	10118	843	p0	578		
3	11	6932	578	p111	578		
4	12	9184	765	p112	765		
5	13	7350	612	p121	765		
6	14	4833	403	p122	765		
7	21	6764	564	p131	612		
8	22	6363	530	p132	612		
9	23	6419	535	p133	612		
10	24	7783	649	p134	612		
11	25	9063	755	p141	403		
4.0		6700		440	400		

Figure 2.12. Wage sheet data format

**Employ\_Dr, Employ \_Ai, Employ \_Plpai, Employ \_VvAPvi, Employ \_VpApZp** (Nodarb\_Dr, Nodarb\_Ai, Nodarb\_Plpai, Nodarb\_VvAPvi, Nodarb\_VpApZp) sheet format:

All these sheets have a unified format. Each sheet presents only one level of education according to the above reflected abbreviations.

Each sheet contains a table that reflects the employed population by age (5 year-old) groups, gender, fields of education, occupations (people). Table starts in B3 cell, the data has been entered, starting from D5 cell.

"Employment" group sheet data format and visual appearance (Print Screen) are presented in Figure 2.13.





Figure 2.13. "Employment" group sheet data format

Active\_V\_str (Aktīvo\_V\_str) sheet format:

The sheet contains a table that reflects the part of economically active population (of total population) by age (5-year-old) groups and gender. The table starts in A1 cell, the data has been entered, starting from C2 cell. The table has a vertical orientation. A column is designed to reflect the age group (in ascending sequence starting from the youngest), B column is designed to reflect the gender (starting from men to women), starting from C column, the data should be entered. First line represents forecasting year.

Figure 2.14 reflects the visual appearance of the table (Print Screen).

	A	В	C	D	E	F	G
1	VG	Dzimums	2010	2011	2012	2013	20
2		Virieši	0,078313	0,078313	0,078313	0,078313	0,0783
3	15_19	Sievietes	0,081026	0,081026	0,081026	0,081026	0,0810
4		Virieši	0,676765	0,676765	0,676765	0,676765	0,6767
5	20_24	Sievietes	0,537455	0,537455	0,537455	0,537455	0,5374
6		Virieši	0,925588	0,925588	0,925588	0,925588	0,9255
7	25_29	Sievietes	0,801910	0,801910	0,801910	0,801910	0,8019
8		Virieši	0,928718	0,928718	0,928718	0,928718	0,9287
9	30_34	Sievietes	0,844646	0,844646	0,844646	0,844646	0,8446
10		Virieši	0,943682	0,943682	0,943682	0,943682	0,9436
11	35 39	Sievietes	0 840719	0 840719	0 840719	0 840719	0 8407

Figure 2.14. Active\_V\_str sheet data format

Study\_place (Stud\_viet) sheet format:

The sheet contains a table that reflects the number of vacant study places by levels and fields of education. The table starts in A1 cell, the data has been entered, starting from C2 cell. The table has a vertical orientation. A column is designed to reflect the name of level of education (starting from Doctor's degree to lower level studies), B column - code of the field of education (in

ascending sequence, starting from the lowest), starting from C column, the data should be entered. First line represents forecasting year.

Figure 2.15 reflects the visual appearance of the table (Print Screen).

	Α	В	С	D	E	F
1	Izgl_lim	Izglītības joma	2010	2011	2012	201:
2		i141	0	0	0	(
3		i142	74	74	74	74
4		i211	0	0	0	(
5		i212	21	21	21	2'
6		i213	0	0	0	(
7		i214	0	0	0	(
8		i215	0	0	0	(
9		i216	0	0	0	(
		1001				

Figure 2.15. Study\_place sheet data format

**Initial\_vac** (sakot\_vak) sheet format:

The sheet contains a table that reflects the number of vacant study places by occupations, education levels, fields and gender. The table starts in A1 cell, the data has been entered, starting from C3 cell.

Initial\_vac sheet data format and visual appearance (Print Screen) are presented in Figure 2.16.

	А	В	С		
1			Gender (starting from men, till women)		Gender D
1			Education field code (in		Education
2			from a smaler)		field code J
3	Occupation code (in growing sequence, starting from a smaler)	Education level (starting from the doctoral level education, till a lower education)			
				Dat	a fields
	Occupation code P	Education level L			

	Α	В	С	D	E	F	G
1			Vīrieši	Vīrieši	Vīrieši	Vīrieši	Vīrieši
2	Profesijas	Izglītības līmeni	j141	j142	j211	j212	j213
3	p0	1DLI	0	0	0	0	
4	p0	2AI	0	0	0	0	
5	p0	3Plpai	0	0	0	0	
6	p0	4VvAPvi	0	0	0	9,974499	
7	p0	5VpApZp	0	0	0	0	
8	p111	1DLI	0	0	0	0	
9	p111	2AI	4,174192	0	0	0	
10	p111	3Plpai	0	0	0	0	
11	p111	4VvAPvi	0	0	0	0	
12	p111	5VpApZp	0	0	0	0	
10	n110	1011	0	0	0		

Figure 2.16. Initial\_vac sheet data format

The table of initial number of vacancies can be filled, using statistical data, or exporting from the original SDM model the calculated number of vacancies (the element "Generated initial number of vacancies" from the Market sheet).

#### Educ\_ind (Izgl\_noz) sheet format:

The sheet contains a table: "The division of employed by educational levels, fields, occupations by sectors" (people). The table starts in A1 cell, the data has been entered, starting from E3 cell.

Educ\_ind sheet data format and visual appearance (Print Screen) is presented in Figure 2.17.

	А	В	С	D	Е		
							Title of
1					Title of occupation		occupation P
					Occupation code (in		
					growing sequence, starting		Occupation
2					with a smaller		code P
	Industry code (in	Education level (starting	Education field code				
	alfabetical order	from doctoral level	(in a growing				
	starting from A	studies, till a lower	sequence, starting	Title of education			
3	to R)	education)	from a smaller)	field			
						Dat	a
			Education field code	Title of education			
	Industry code N	Education level L	J	field J			

1					Specializê	Viesmīlība	Mazumtin	Citu pakal	Zinātnieki	Matemāti	Dabas zini	Inžen
2	Nozare	Izglītības pakāpe	Izglītības jomas	Izglītības jomas nosaukums	134	141	142	143	211	212	213	214
9			i215	Amatniecība								
10			i216	Lietišķāmāksla								
11			i221	Reliģija un teoloģija								
12			i222	Valodu un kultūras studijas								
13			i223	Dzimtās valodas studijas					163			
14			i224	Vēsture, filozofija un radniecīgā					234			
15			i310	Sociālās un cilvēkrīcības zinātn								
16			i321	Žurnālistika un komunikācija								
17			i322	Bibliotēku, informācijas un arhīv								
18			i341	Vairumtirdzniecība un mazumti								

Figure 2.17. Education sheet data format

Educ\_ind sheet is a logically organized table. The size of the table (matrix) exceeds the Powersim software restrictions in relation to the matrix sizes. SDM model takes the data from this table, virtually dividing the table into two parts: the sectors with codes from A to J, and from K to R.

## Mathematical formula abbreviations

AAK - wage ratio; ABS<sub>JD</sub> - number of applicants by gender and fields of education; ABSM<sub>JD</sub> - number of applicants by gender and fields of education in lifelong education; ABSS<sub>JD</sub> - structure of applicants and fields of education of graduates; ABStM<sub>JD</sub> - structure of alumni's and fields of education of graduates in lifelong education; AbsVSt<sub>Vg</sub> - age structure of alumni; ADAVS<sub>DPJ</sub> - adapted number of vacancies; ADV<sub>DPI</sub> - occupied working places: ADVK<sub>DPJ</sub> -correction of occupied working places; ADVS2<sub>DPJ</sub> - decrease in the number of occupied working places in the 2nd priority; ADVS<sub>DPJ</sub> - decrease in the number of occupied working places; AIAM<sub>PI</sub> - integrated education compliance matrix. AI<sub>J</sub> - learning time; AIM<sub>VgDPJE</sub> - return immigration; AK - level of education dropout index; AKs - women dropout index; A<sub>L</sub> - average wages at the levels of education; AM<sub>VgDPJE</sub> - change of the group of the economic activity of the population; AMVGM<sub>VgDPJE</sub> - change of the economic activity group along with the change of age; AP<sub>GJD</sub> - learning process; ASM<sub>VgDPJ</sub> - change of economic activity status for active population; ASP<sub>VgDPJe</sub> - assignment of the status of economic activity for the active population; AtDV<sub>DPJ</sub> - number of remaining working places; AtDVS<sub>DPJ</sub> - structure of remaining working places;  $AT_{GJD}$  - dropout; ATL<sub>DPJ</sub> - surplus; ATS1<sub>VgDPJ</sub> - release from work in the 1st priority; ATS2<sub>VgDPJ</sub> - release from work in the 2nd priority;  $ATSS_{Vg}$  - privileged age structure of the release from work; ATVS<sub>DPJ</sub> - number of appropriate vacancies; ATVSSt<sub>DPJ</sub> - structure of appropriate vacancies; AvpNap<sub>GID</sub> - change of general secondary education to the vocational education with the basic education; AVSA<sub>DPJ</sub> - structure of remaining vacancies; B<sub>GID</sub> - admission in the base period; BI<sub>VID</sub> -15-year-old population without education; BK - bachelor's index;  $B_{VJD}$  - bachelor degree or qualification; D - gender; D<sub>D</sub> - birth rate;  $D_{DV}$  - the number of born people by gender and maternal age; DK<sub>DSV</sub> - birth rate index; DKI<sub>DSV</sub> - change of birth rate index; DKND - independent part of birth rate index; DKPT - total growth rate of birth rate; DKPTiIPT - limitation of total birth rate index growth in accordance with the long-time growth rate; DKPTiMR - limitation of total birth rate index growth in accordance with the maximum threshold; DKPTIP - growth rate of birth rate in the long-term period;

DKRK - regression index of the birth rate index; DKS - birth rate per woman of reproductive age; DKSM - maximum total birth rate per woman; DKSPA - total birth rate per woman of reproductive age in case of GDP growth; DKSSA - total birth rate per woman of reproductive age in case of GDP reduction;  $DM_D$  - number of jobseekers by gender; DMIs<sub>VgDPJ</sub> - ratio of jobseekers in active population; DM<sub>L</sub> - number of jobseekers by levels of education;  $DMN^{1}_{VgDPJ}$  - occupation of vacancies in the *i* priority; DMNKOR<sub>VgDPJ</sub> - correction of the number of jobseekers and employed; DMN<sub>VgDPJ</sub> - move from job search to employment;  $DM_P$  - number of jobseekers by occupations; DMP<sub>VgDPJ</sub> - increase of the number of jobseekers; DMSP<sub>VgDPJ</sub> - forecast of the number of jobseekers; DMS<sub>VgDPJ</sub> - decline in the number of jobseekers; DM<sub>VgDPJ</sub> - number of jobseekers;  $DM_{VgDPJ i}$  - number of jobseekers at *i* level of education; DMvS<sub>VgDPJ</sub> - change of the age structure of jobseekers; DV<sub>DPJ</sub> - number of working places; DVI<sub>DPJ</sub> - working place is being occupied; DVIN<sub>DPJ</sub> - working place is being occupied in accordance with the occupation of the vacancy; DVIX<sub>DPJ</sub> - working place is being occupied in accordance with the vacancies; DVJ<sub>DPJ</sub> - creation of new working places; DVLaB<sub>DPJ</sub> - appropriate decrease of number of working places; DVLaD<sub>DPI</sub> - adapted decrease of number of working places; DVL<sub>DPJ</sub> - liquidation of working places; DVT<sub>DPJ</sub> - working place is being released; E - economic activity group; EAIM<sub>VgDPJ</sub> - change of economic activity group of the population in accordance with the target; EAIP<sub>VgDPJ</sub> - growth of active population;  $EAISt_{VgDL}$  - structure of economically active population by levels of education, age groups and gender. EAISt<sub>VgDPJ</sub> - structure of economically active population by age groups, gender, occupations and fields of education; EAI<sub>VgDL</sub> - economically active population by levels of education, age groups and gender;  $EAI_{V_{g}DPJ}$  - economically active population; EEVK - economic migration formation index; EIM<sub>V<sup>o</sup>DPJE</sub> - economic immigration; EIVK - economic immigration formation index;  $EMES_{V \in DPJE}$  - labour immigration in accordance with the current structure; EMUS<sub>VgDPJE</sub> - labour immigration in accordance with the provided structure;  $EM_{VD}$  - emigration; ENIP<sub>VgDPJ</sub> - growth of inactive population; ET - emigration propensity; G - learning year; GAAD - readiness to invite the foreign employees; i - birth rate distribution time; IA<sub>GID</sub> - mastering of education; IA<sub>GJD1</sub> - mastering of basic general education or vocational education with general education; IAM - education compliance matrix;

IAMSL<sub>JLP</sub> - structure of education compliance matrix by levels of education;

IAMSM<sub>JLP</sub> - structure of education correlation matrix;

 $Iap_{GJD}$  - mastering of vocational education with general education;

II<sub>GJD</sub> - acquisition of education;

IINDV - longstanding vacant working places of the previous period;

 $II_{VJD}$  - education attainment by age groups;

IKPI - total GDP per capita;

IKP<sub>N</sub> - GDP forecasts by sectors;

 $IKP_{N0}$  - GDP by sectors in the base period;

IKPPT - growth rate of total GDP per capita;

 $ILMK_{VD}$  - balanced death rates in the long-term period;

ILSITK - balanced education continuation index in the long-term period;

 $IMES_{VgDPJE}$  - labour immigration in accordance with the current structure;

IMITPR - immigration ratio for solution of the labour shortage problem;

IMSt<sub>Vg</sub> - age structure of immigrants;

IM<sub>VD</sub> - immigration;

IMVNV - method of definition of immigration types;

INDV - longstanding vacant working places;

INDVD - definition of longstanding vacant working places;

INDVI - changes of longstanding vacant working places;

INDVM - longstanding vacant working places, which are forming immigration;

INDVSt<sub>DPJ</sub> - structure of longstanding vacant working places;

IPa<sub>J</sub> - demand of fields of education for the vocational education and vocational secondary education;

IPIKPI - total GDP per capita in the previous period;

IP<sub>J</sub> - structure of field of education demand;

 $ISI_{VgD}$  - population ratio of the total population by age groups and gender;

 $ISPK_N$  - short-term expected productivity index by sectors;

ISSI<sub>VgDPJE</sub> -change of population and structure;

ISt<sub>VgDPJE</sub> - population structure;

ITK - education continuation index;

ITKi - change of education continuation index;

ITKiAL - change of education continuation index, in accordance with the wage level;

ITKiIT - change of education continuation index, in accordance with the long-term trends;

ITKiNL - change of education continuation index, in accordance with the employment rate;

I<sub>VD</sub> - population;

 $I_{VgD}$  - population by age groups and gender;

 $I_{VgDPJE}$  - population;

IZMSt<sub>VgDPJE</sub> - structure of maximum of changes;

 $IZM_{VgDPJE}$  - maximum of changes;

J - field of education;

 $JJVS_J$  - initial number of study places in the new fields;

JSVA<sub>J</sub> - opening of study places in the new fields;

KDKI - choice of the total birth rate index;

KDM<sub>DPJL</sub> - total number of jobseekers;

KIJ<sub>J</sub> - critical fields of education;

KI<sub>VgD</sub> - total population by age groups and gender;

KLSI<sub>PN</sub> - changes of total labour demand by occupations and sectors;

KN<sub>DPJL</sub> - total number of employed;

KOR<sub>VgDPJ</sub> - correction;

KVS<sub>DPJL</sub> - total number of vacancies;

L - level of education (group);

LMSt<sub>PN</sub> - target structure of the labour demand by occupations and sectors;

 $LSA_{JLP}$  - labour demand in the occupation with the appropriate education by fields and levels of education;

 $LS_D$  - labour demand by gender;

LSIA - average ratio of labour demand and active population;

LSIA<sub>J</sub> - average ratio of labour demand and active population by fields of education;

LSIENK - estimation index of the stages of changes of labour demand;

 $LSIMSt_{PN}$  - changes of labour demand by occupations and sectors in accordance with the sectorial changes on the basis of the target structure;

LSI<sub>N</sub> - changes of labour demand by sectors.

LSINLSI<sub>PN</sub> - changes of labour demand by occupations and sectors in accordance with the sectorial changes of labour demand.

 $LSI_P$  - changes of labour demand by occupations.

LSI<sub>PN</sub> - changes of labour demand by occupations and sectors;

LS<sub>JLDP</sub> - labour demand by fields of education, levels, occupations and gender;

 $LS_{JLP}$  - labour demand by fields, levels of education and occupations;

LSMSt<sub>PN</sub> - labour demand by occupations and sectors on the basis of the target structure;

 $LS_N$  - labour demand by sectors;

 $LS_{N0}$  - labour demand by sectors in the base period;

LS<sub>P</sub> - labour demand by occupations;

LSPA<sub>JLP</sub> - growth of labour demand by fields, levels of education and occupations, in accordance with the compliance of education to occupation;

LSPA<sub>P</sub> - growth of labour demand by occupations, in accordance with the compliance of education to the requirements of occupation;

LSP<sub>DPJ</sub> - growth of labour demand by occupations and gender at the level of education;

LSP<sub>JLDP</sub> - growth of labour demand by fields, levels of education and occupations and gender;

LSP<sub>JLP</sub> - growth of labour demand by fields, levels of education and occupations;

LSPL<sub>JLP</sub> - growth of labour demand by fields, levels of education and occupations, in accordance with the structure of labour demand by levels of education;

 $LSPL_P$  - growth of labour demand by occupations, in accordance with the structure of labour demand by levels of education;

LSPM<sub>JLP</sub> - growth of labour demand by fields, levels of education and occupations, in accordance with the structure of the educational compliance matrix;

 $LS_{PN}$  - growth of labour demand by occupations and sectors;

LSPnAL<sub>P</sub> - growth of labour demand by occupations, without regard to the compliance of education to the requirements of occupation and structure of labour demand by levels of education;

 $LSPnA_P$  - growth of labour demand by occupations, without regard to the compliance of education to the occupation;

LSP<sub>P</sub> - growth of labour demand by occupations;

LSS<sub>JLDP</sub> - decrease of labour demand by fields, levels, of education occupations and gender;

 $LSS_{JLP}$  - decrease of labour demand by fields, levels of education and occupations;

LSS<sub>P</sub> - decrease of labour demand by occupations;

 $LSStA_{JLP}$  - structure of labour demand in the occupation with the appropriate education by fields and levels of education;

LSSt<sub>DP</sub> - structure of labour demand by occupations and gender;

LSSt<sub>JLP</sub> - structure of labour demand by fields, levels of education and occupations;

 $LSSt_{LP}$  - structure of labour demand by levels of education and occupations;

 $LSSt_{PN}$  - structure of labour demand by occupations and sectors;

MAAK - target wage ratio;

 $MEAI_{VgD}$  - target economically active population by age groups and gender at i level of education;

 $MEAI_{VgDL}\xspace$  - target economically active population by levels of education, age groups and gender;

MEAI<sub>VgDPJ</sub> - target economically active population;

MI<sub>VD</sub> - migration;

 $MKN_{VD}$  - death rate variance;  $MK_{VD}$  - death rate. MR - mortality response to changes of GDP;  $M_{VD}$  - mortality; M<sub>VgD</sub> - mortality by age groups and gender in 15-70-year-old groups; M<sub>VgDPJE</sub> - mortality; N - sector: NAVS<sub>P</sub> - surplus of occupied working places; NDM<sub>VgDPJ</sub> - move from employment to job search; NEAIM<sub>VgDPJ</sub> - negative change of economic activity group of the population in accordance with the target: NKOR<sub>VgDPJ</sub> -correction of the structure of employed; N<sub>LJ</sub> - number of employed by levels and fields of education; NLSI<sub>N</sub> - necessary labour changes by sectors; NLS<sub>N</sub> - necessary labour by sectors; NMI2<sub>VgDPJ</sub> - maximum possible changes of employed from LS;  $NMI2_{V \oplus DPJ}$  - number of employed in the 2nd priority of release from work; NMI<sub>VgDPJ</sub> - maximum possible changes of employed;  $N_{N0}$  - employment by sectors in the base period; NNKOR<sub>VgDPJ</sub> - required correction of the structure of employed; NOVRb<sub>VgDPJE</sub> - aging result without the economic activity groups;  $NOVR_{VgDPJE}$  - aging result;  $NOV_{VD}$  - aging;  $NOV_{VgD}$  - aging by age groups and gender in 15-70-year-old groups; NOV<sub>VgDPJE</sub> - aging; N<sub>P</sub> - number of employed by occupations; NPIL - time of the change of employment; NPKI<sub>N</sub> - the necessary changes of productivity index by sectors; NPK<sub>N</sub> - necessary productivity index by sectors; NPMA<sub>VgDPJ</sub> - change of occupation; NPM<sub>VgDPJ</sub> - change of occupation of employed; NPMV<sub>VgDPJ</sub> - occupation of vacancies of employed;  $NP_{VgDPJ}$  - decrease in the number of employed; NS<sub>DPJ</sub> - decrease in the number of employed due to the change of age structure by occupations, fields of education and gender; NSP<sub>VgDPJ</sub> - forecast of the number of employed;  $NS_{V_{0}DPI}$  - decline in the number of employed due to the change of age structure; NTMa<sub>GID</sub> - decide to continue the studies in vocational education (vocational secondary education); NTMpp<sub>GJD</sub> - decide to continue the studies after basic education; NTMv<sub>GID</sub> - decide to continue the studies in general secondary education;  $NTM_{VID}$  - decide to continue the studies;  $N_{VgD}$  - number of employed by age groups and gender;  $N_{VgDPJ i}$  - number of employed at *i* level of education;  $NvS2_{Vg}$  - age structure of employed in the 2nd priority;  $NvS_{VgDPJ}$  - change of the age structure of employed; OIINDV - longstanding vacant working places of the second level in the previous period; OSVIJL - change of study places; OSV<sub>J</sub> - number of optimized study places; OVSV<sub>J</sub> - optimized vacant study places; P - occupation; PEAIM<sub>VgDPJ</sub> - positive change of economic activity group in accordance with the target;

MKI - change of death rate in accordance with GDP growth;

 $PIL_N$  - time of the change of productivity by sectors;

PIM<sub>VgDPJE</sub> - demanded immigration;

PITKiAL - changes of demanded education continuation index in accordance with the wage level; PITKiNL - changes of demanded education continuation index in accordance with the level of employment;

PKI<sub>N</sub> - changes of productivity index by sectors;

 $PK_{N0}$  - labour productivity index by sectors in the base period;

 $PMDM_{VgDPJ}$  - change of occupation in order to get a job;

 $PMN_{VgDPJ}$  - employment after the change of occupation;

 $PMTB_{VgDPJE}$  - change of occupation in the market module;

PNNKOR<sub>VgDPJ</sub> - additionally required correction of employed structure;

PP<sub>PJ</sub> - occupational demand in accordance with IAM;

 $PPS_{PJ}$  - demand structure of occupation in accordance with IAM for the population with general secondary and vocational education;

 $PPS_{PJ}$  - structure of occupational demand in accordance with IAM;

PS - number of applications;

PSK - measure of importance of occupation;

PSSVA - total ratio of number of applications and study places;

PSt<sub>Vg</sub> - preferential age structure;

 $Pvp_{GJD}$  - abandon basic education;

 $RK_N$  - productivity index by sectors;

SA<sub>GJD</sub> - beginning of education mastering;

SLSI - the changes of the major occupations of the labour demand;

SMM<sub>GD</sub> - mortality and migration of students;

SP<sub>GJD</sub> - interruption of education;

SPKI<sub>N</sub> - total ratio of major occupations by sectors;

SPmm<sub>GJD</sub> - interruption of education due to mortality or migration;

SR<sub>V</sub> - number of women of reproductive age;

SS<sub>J</sub> - student structure by fields of education;

SUSV<sub>J</sub> - summed up advanced study places;

SVIR - limit of changes of study places;

SVP<sub>J</sub> - private financing of study places;

t - forecasting time (forecasting current year);

t<sub>b</sub> - forecasting time horizon (forecasting last year);

TILS - time to get integrated into Latvian society;

t<sub>ITK</sub> - period of change of education continuation index;

TML - time to take the emigration decision;

TVSM - time for occupation of vacancies for migration;

UEAI<sub>VgD</sub> - provided economically active population;

ULL<sub>VgD</sub> - provided participation level;

ULSI - accumulated changes of labour demand;

ULSIN - estimation of the accumulated changes of labour demand;

UN<sub>VgD</sub> - removal from analysis by age groups and gender;

UN<sub>VgDPJE</sub> - removal from analysis;

UP<sub>VgDPJE</sub> - admission for analysis;

USV<sub>J</sub> - advanced study places;

V1NKOR<sub>VgDPJ</sub> - age group correction 1;

V2NKOR<sub>VgDPJ</sub> - age group correction 2;

VAGA<sub>VgDPJE</sub> - leaving the old analysis group;

Vg - age group;

VS<sub>D</sub> - number of vacancies by gender;

VS<sub>DPJ</sub> - number of vacancies;

 $VS_{DPJ_i}$  - number of vacancies at the education level *i*;

 $VS_L$  - number of vacancies by levels of education;

VS<sub>P</sub> - number of vacancies by occupations;

 $VSS_{DPJ}$  - decline in the number of vacancies;

VSVA<sub>JD</sub> - occupation of vacant study places;

VSVAM<sub>JD</sub> - occupation of vacant study places in the lifelong education;

VSV<sub>J</sub> - vacant study places.

# Data recovery time.

No.	SDM_in file	Data name	Data source	Update	
	sheet name				
1.	Employment / Nodarbinātība	Employment by sectors in the base year	CSB OVERVIEW OF WAGE STRUCTURE (5-work); OVERVIEW OF WORK (3-work)	75days after the end of the year of account	
2.	GDP / IKP	GDP forecast	МоЕ	Upon preparedness of the forecast	
3.		GDP in the base year	CSB overview table IK04. GROSS DOMESTIC PRODUCT BY TYPE OF ACTIVITY	70 after the end of the year of account	
4.	Indexes / Koeficienti	Time index of changes of labour productivity of the sectors	Experts	Over the period of GDP base year update the examination should be carried out, whether it is required to update. If necessary, it should be updated along with GDP.	
5.	Occupations / Profesijas	Division of employed by sectors and occupations in the base year	CSB OVERVIEW OF WAGE STRUCTURE (5-work); OVERVIEW OF WORK (3-work)	75days after the end of the year of account	
6.		Target labour sectorial- occupational structure	MoE	Upon completion	
7.	Education / Izglītība	Division of employed by levels of education, fields, occupations	CSB OVERVIEW OF WAGE STRUCTURE (5-work); OVERVIEW OF WORK (3-work)	75days after the end of the year of account	
8.		Occupational compliance matrix	MoE	Does not require regular updates. Should be reviewed once in 5- 10 years.	
9.	Demography / DemogrāfijaPopulation gender in the base year.CSB STRUCTURE OF AT THE BEGINNING OF THE YEAR		13.06.2013.		
10.		Population mortality by age and gender	CSB overview table IM01. DEATHS BY AGE AND SEX	03.07.2013.	
11.		Alive born people by age of the mother	CSB overview table ID02. NUMBER OF BORN BY GENDER and ID03. ALIVE BORN PEOPLE BY AGE OF MOTHER	03.07.2013.	
12.	Dem_Dr	Population by levels of	CSB OVERVIEW OF WAGE	75 days after the end of	
13.	Dem_Ai	education in the base year	SIRUCTURE (5-work); OVERVIEW OF WORK (3-work)	the year of account	
14.	Dem_Plpai		WORK (S-WORK)		
15.	Dem_VvAPvi				
16.	Dem_VpApZp			01.02.2012	
17.	Primary_pupil s / Sak_skoln	Number of pupils in the base year	CSB overview table IZG09. NUMBER OF EDUCATED IN GENERAL ACADEMIC SCHOOLS BY CLASSES (at the beginning of the school year)	01.03.2013.	
18.	Students / Studējošie	Mastering of vocational education with the basic education	OVERVIEWS PROF-2 and PROF-2m OF VOCATIONAL EDUCATIONAL INSTITUTIONS	21.06.2013.	
19.		Mastering of vocational education and vocational secondary education		21.06.2013.	
20.		Mastering of first level higher vocational education		21.06.2013.	

r				
21.		Mastering of bachelor's degree		21.06.2013.
22.		Mastering of master's		21.06.2013.
23		Mastering of doctor's degree		21.06.2013.
23.	Admission /	Vocational education and	OVERVIEWS PROF-2 and PROF-2m OF	21.06.2013.
27.	Uzņemšana	vocational secondary education	VOCATIONAL EDUCATIONAL INSTITUTIONS	
25.		First level higher vocational education		21.06.2013.
26.		Bachelor's degree		21.06.2013.
27.		Master's degree		21.06.2013.
28.		Doctor's degree		21.06.2013.
29.	Education_sup ply / Izglītības_pied	Duration of the studies	MoE	Does not require regular updates. Should be reviewed once in 3- 7 years.
30.	Additional matrix / Papildmatrica	Additional matrix	-	Does not require updates
31.	Salaries / Algas	Monthly salaries in the base year	CSB OVERVIEW OF WAGE STRUCTURE (5-work); OVERVIEW OF WORK (3-work)	65 day after the end of the quarter of account
32.	Employ_D /	Employed population in the	CSB OVERVIEW OF WAGE	75days after the end of
33.	Nodarb_D Employ_Ai / Nodarb_Ai	base year	STRUCTURE (5-work); OVERVIEW OF WORK (3-work)	the year of account
34.	Employ_Plpai			
	, Nodarb_Plpai			
35.	Employ_VvA			
	Pvi /			
	Nodard_VVAP			
36.	Employ_ VpApZp / Nodarb_VpAp Zp			
37.	Active_V_str / Aktīvo_V_str	Part of economically active population	MoE	Upon completion
38.	Study_places / Stud_viet	Number of new state- financed study places	MoE	Dependence on the simulation target. May not be used or use previous simulation results
39.	Initial_vac / Sakot_vak	Number of vacancies in the base year	Generated in SDM.sip file; data can be obtained in another way (statistically or from experts).	Generated in SDM.sip file, it is recommended to be updated before each simulation
40.	Educ_sector / Izgl_noz	Division of employed by levels of education, fields, occupations and sectors in the base year.	CSB OVERVIEW OF WAGE STRUCTURE (5-work); OVERVIEW OF WORK (3-work)	75days after the end of the year of account

Annex A.10.

## Data recovery and forecasting time consumption.

No	Activity group Activity		Time	Total
•				group
1.		Update of the table "Employment by sectors in the base year"	10 min	ume
2.		Update of the table "GDP in the base year"	10 min	
3.		Update of the table "Division of employed by sectors and occupations in the base year"	15 min	
4.		Update of the table " Division of employed by education levels, fields and occupations"	20 min	
5.		Update of the table " Population by age and gender in the base year"	10 min	
6.		Update of the t] able "Population mortality by age and gender"	10 min	
7.	Data update	Update of the table " Alive born people by age of the mother"	15 min	13
8.	-	Update of the tables "Population by levels of education in the base year" for the five levels of education together	240 min	nours
9.		Update of the table " Number of pupils in the base year"	10 min	
10.		Update of the tables "Mastering of education" for the five levels of education together	60 min	
11.		Update of the tables "Admission by levels of education" for the five levels of education s together	60 min	
12.		Update of the table "Monthly salaries in the base year"	30 min	
13.		Update of the tables "Employed population in the base year" for the five levels of education together	240 min	
14.		Update of the table "Division of employed by education levels, fields, occupations and sectors in the base year"	30 min	
15.		Review of the GDP forecast	5 min	
16.	Assumption data	Review of the target labour sectorial-occupational structure	30 min	2
17.	review	Review of the part of the economically active population	30 min	hours
18.		Review of the number of state-financed new study places	30 min	
19.		Review of the duration of the studies	30 min	
20.	Special data	Review of the occupational compliance matrix	480 min	10
21.	review (option)	Review of the time indexes of the sectorial labour productivity	30 min	hours
22.	Implementation	Modelling mode setup	5 min	0.2
23.	of pre-modelling procedure	of pre-modelling Occupation of number of vacancies in the base year procedure		hours
24.	Modelling process	delling  Modelling process (one simulation)    cess		0,2 hours
25.	Model full 5 simulations for fitting of each constant (number of constants 58)		10 min	48 hours
26.	Result examination	Result examination	10 min	0,2 hours